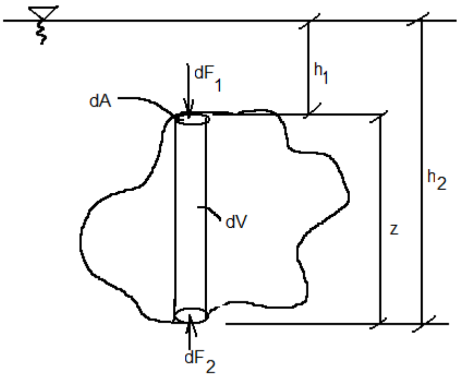
FLUID MECHANICS

BUOYANCY- The Lifting Force Applied To Submerged Object

As shown in figure, let's take a unit area in upper and lower surface of object. The difference of pressure forces acting on these surfaces as fallows.



$dF = dF_2 - dF_1 = P_2\,dA - P_2\,dA$

$dF = (\varrho\, g\, h_2\, dA) - (\varrho\, g\, h_1\, dA)$

$dF = \varrho\, g\, (h_2 - h_1) dA$ $\{dV = (h_2 - h_1) dA\}$

$dF = \varrho\, g\, dV$

$F = \int \varrho\, g\, dV$

$\boxed{F = \varrho\, g\, V_{submerged}}$

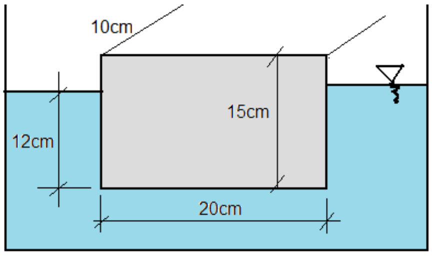
Here the unit volume is scanned the entire volume, the lifting force can be obtained acting the object. This force is equal to the weight of the fluid of filling the submerged volume of the object. This is called the lifting force (or Archimedes principle).

Briefly, the **Buoyant Force, is equal to the weight of the fluid to the volume of the submerged part of the object**.

The forces applied the side surfaces are equal and opposite direction, there for no force act from side surfaces.

Lifting force application point is the submerged volume center (pressure center).

Example: As shown in the figure, a prism is left in the water. Object is floating. Find the weight of this object. Prism sizes 20x15x10 cm. The height of the sinking portion is 12 cm.



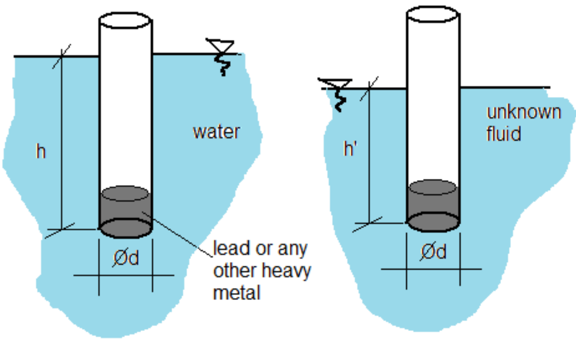
If body is swimming => { Body Weight } = { Lifting Force }

$G = F_{lift} = \varrho\, g\, V_{submerged}$

$= 1000\ kg/m^3\ 9.81\ m/s^2\ (0.2m\ 0.12m\ 0.1m)$

$= 23.54\ (kgm/s^2 = N) = 2.4\ kgf$

Example: The density of liquids is measured by hydrometer device (See figure). Making a hydrometer with a cylinder shaped container (pen container may be), we want to measure the density of a liquid but density would not know. How do we measure?



$G = F_{lift} = \varrho\, g\, V_{submerged}$

Both are floats, so lifting forces equal to the weight of the object.

ϱ_{water} g (πD^2 h/4) = $\varrho_{unknown}$ g (πD^2 h`/4)

ϱ_{water} h = $\varrho_{unknown}$ h'

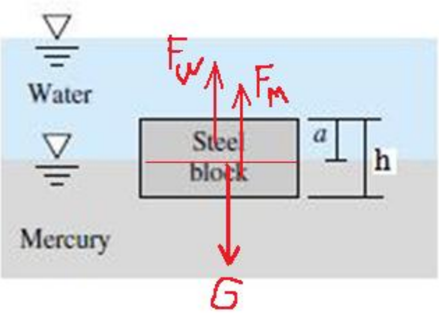
$\varrho_{unknown}$ = ϱ_{water} h / h'

For example: h = 10 cm, h = 12 cm, get. Density is as follows.

$\varrho_{unknown}$ = ϱ_{water} h / h'= 1000 kg/m^3 0.1 m/ 0.12m = 833.3 kg/m^3

4) A uniform block of steel (density= 7,6) will float at a mercury-water interface as in Figure. What is the distance a for this condition h=25cm? (mercury density=13,81)

d1 = 7,6 * 1000 kg/m3
d2 =13,81 *1000 kg/m3
d3 = 1* 1000 kg/m3
h = 25cm
a=?



G = d1 * g* (L? * W? * h)
Fw = d3* g* (L? * W? * a)
Fm = d2* g* (L? * W? * (h-a))

G = Fw + Fm
d1 * g* (L? * W? * h) = d3* g* (L? * W? * a) + d2* g* (L? * W? * (h-a))
d1 * h = d3*a + d2* (h-a)
d1 * h = d3*a + d2* h - d2* a
d1 * h = a(d3 –d2) + d2* h
(d1 * h)- (d2* h) = a(d3 –d2)

a= ((d1 * h)- (d2* h)) / (d3 –d2)
a= ((7600 * 0,25)- (13810* 0,25)) / (1000 –13810) (Note: We can use relative density here, Then it would be much simpler)
a= ((1900)- (3452,5)) / (-12810)
a= (-1552,5) / (-12810) =0,121 m
a= 0,121 m =12,1 cm

STABILITY

Occasionally, a body will have exactly the right weight and volume for its ratio to equal the specific weight of the fluid. If so, the body will be *neutrally buoyant* and will remain at rest at any point where it is immersed in the fluid. A submarine can achieve positive, neutral, or negative buoyancy by pumping water in or out of its ballast tanks.

A floating body as in Figure may not approve of the position in which it is floating. If so, it will overturn at the first opportunity and is said to be statically *unstable*. The least disturbance will cause it to seek another equilibrium position which is stable. Engineers must design to avoid floating instability. The steps are as follows:

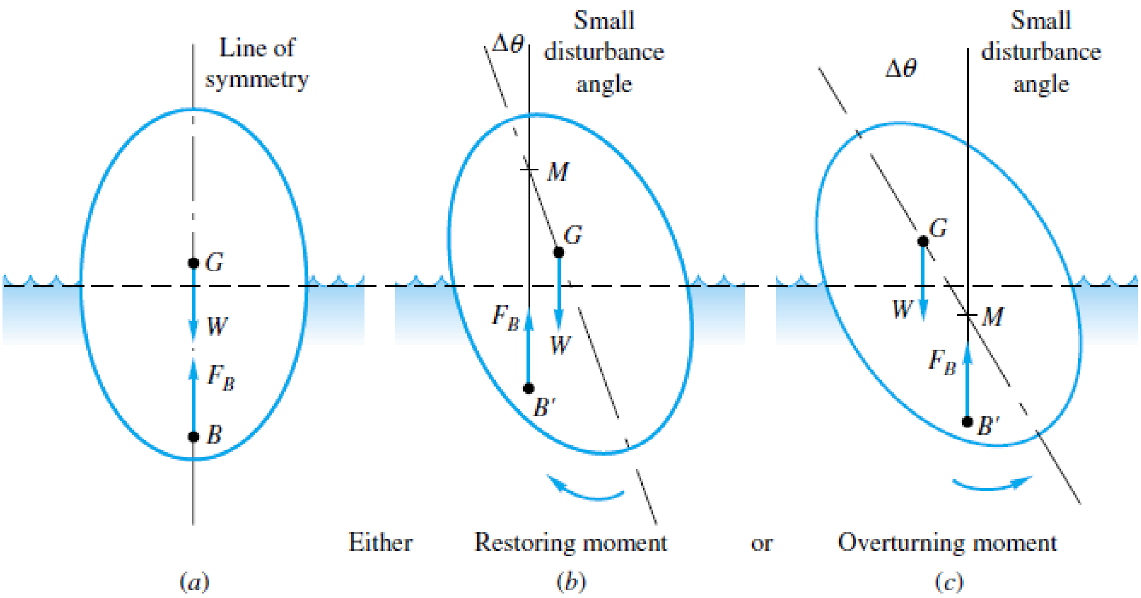


Figure. Calculation of the metacenter *M* of the floating body shown in (*a*). Tilt the body a small angle Δθ Either (*b*) *B*' moves far out (point *M* above *G* denotes stability); or (*c*) *B*' moves slightly (point *M* below *G* denotes instability).

1. The basic floating position is calculated from the Equation which will come next topic. The body's center of mass *G* and center of buoyancy *B* are computed.

2. The body is tilted a small angle Δθ, and a new waterline is established for the body to float at this angle. The new position *B*' of the center of buoyancy is calculated. A vertical line drawn upward from *B*' intersects the line of symmetry at a point *M*, called the *metacenter*, which is independent of Δθ for small angles.

3. If point *M* is above *G*, that is, if the *metacentric height* $\overline{MG}$ is positive, a restoring moment is present and the original position is stable. If *M* is below *G* (negative $\overline{MG}$, the body is unstable and will overturn if disturbed. Stability increases with increasing $\overline{MG}$.

Thus the metacentric height is a property of the cross section for the given weight, and its value gives an indication of the stability of the body. For a body of varying cross section and draft, such as a ship, the computation of the metacenter can be very involved.

Stability Related to Waterline Area

Naval architects have developed the general stability concepts from Figure into a simple computation involving the area moment of inertia of the *waterline area* about the axis of tilt. The derivation assumes that the body has a smooth shape variation (no discontinuities) near the waterline and is derived from Figure.

The *y*-axis of the body is assumed to be a line of symmetry. Tilting the body a small angle θ then submerges small wedge *Obd* and uncovers an equal wedge *cOa*, as shown.

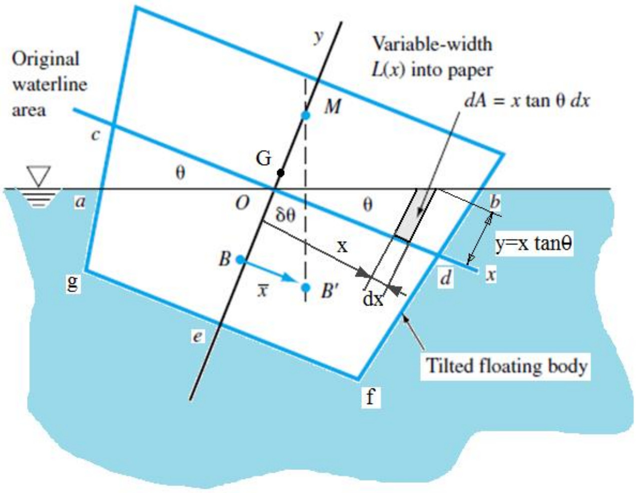


Figure A floating body tilted through a small angle θ. The movement $\bar{x}$ of the center of buoyancy *B* is related to the waterline area moment of inertia.

The new position *B*' of the center of buoyancy is calculated as the centroid of the submerged portion of the body:

Lets take V_{cdfg}, V_{coa}, V_{bod}, V_{abfg} volumes. If the calculated, centers of gravity torques of these volumes according to the y-axis, we find the $\bar{x}$ distance.

$$\bar{x}\,V_{abfg} = \int_{cdfg} x dV + \int_{bod} x dV - \int_{coa} x dV$$

dA= x tanθ dx => dV = L dA

$$= 0 + \int_{bod} x\, L\, dA - \int_{coa} x\, L\, dA$$

$\int_{cdfg} x dV = 0$ has a symmetrical shape. The center of gravity on a line of torque. $\bar{x}$ =0

$$\bar{x}\,V_{abfg} = \int_{bod} x\, L\, x\, tan\theta\ dx - \int_{coa} x\, L\, (-x\, tan\theta\ dx)$$

$$= tan\theta \text{ L } [\int_{bod} x^2\ dx + \int_{coa} x^2\ dx]$$

Detail here

$$= tan\theta \text{ L } [\int_{waterline} x^2\ dx]$$

$dA_{waterline}$ = L dx

$$= tan\theta \int_{waterline} x^2 dA_{waterline}$$

$I_o = \int_{waterline} x^2 dA_{waterline}$

$$\bar{x}\,V_{abfg} = tan\theta\ I_o$$

$$\bar{x} = tan\theta\ I_o / V_{abfg}$$

Let us consider the triangle MBB'

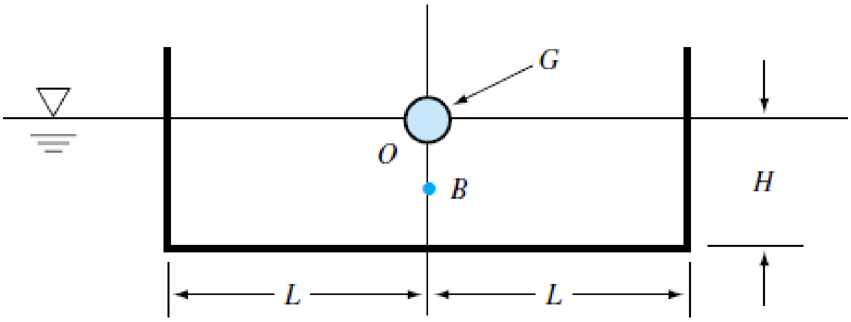
$$Tan\theta = \frac{\bar{x}}{MB} \Rightarrow MB = \frac{\bar{x}}{Tan\theta} \Rightarrow MB = \frac{\frac{Tan\theta\, I_o}{V_{abfg}}}{Tan\theta} \Rightarrow \boxed{MB = \frac{I_o}{V_{submerged}}}$$

$V_{abfg} = V_{submerged}$

$$MB = MG + GB \Rightarrow \boxed{MG = MB - GB} \Rightarrow \boxed{MG = \frac{I_o}{V_{submerged}} - GB}$$

where I_o is the area moment of inertia of the *waterline footprint* of the body about its tilt axis *O*. The engineer would determine the distance from *G* to *B* from the basic shape and design of the floating body and then make the calculation of *Io* and the submerged volume υ_{sub}. If the metacentric height MG is positive, the body is stable for small disturbances. Note that if $\overline{GB}$ is negative, that is, B is *above* G, the body is always stable.

Example: A barge has a uniform rectangular cross section of width $2L$ and vertical draft of height H, as in Figure. Determine (a) the metacentric height for a small tilt angle and (b) the range of ratio L/H for which the barge is statically stable if G is exactly at the waterline as shown.



a) If the barge has length b into the paper, the waterline area, relative to tilt axis O, has a base b and a height $2L$; therefore, $I_O = b(2L)^3/12$. Meanwhile, $\upsilon_{sub} = 2LbH$. Equation predicts

$$\overline{MB} = \frac{I_0}{V_{sub}} = \frac{b(2L)^3/12}{(2L)b\,H} = \frac{L^2}{3H}$$

$$\overline{MG} = \overline{MB} - \overline{GB}$$

$$\overline{MG} = \frac{L^2}{3H} - \frac{H}{2}$$

b) The barge can thus be stable only if

$$\overline{MB} = \overline{GB}$$

$$\frac{L^2}{3H} = \frac{H}{2}$$

$L^2 = 3H^2/2$

$L = 1{,}22\,H$

or $2L > 2.45H$

The wider the barge relative to its draft, the more stable it is. Lowering G would help also.

Even an expert will have difficulty determining the floating stability of a buoyant body of irregular shape. Such bodies may have two or more stable positions. For example, a ship may float the way we like it, so that we can sit upon the deck {güverte}, or it may float upside down (capsized).

Floating instability occurs in nature. Living fish generally swim with their plane of symmetry vertical. After death, this position is unstable and they float with their flat sides up. Giant icebergs may overturn after becoming unstable when their shapes change due to underwater melting. Iceberg overturning is a dramatic, rarely seen event.