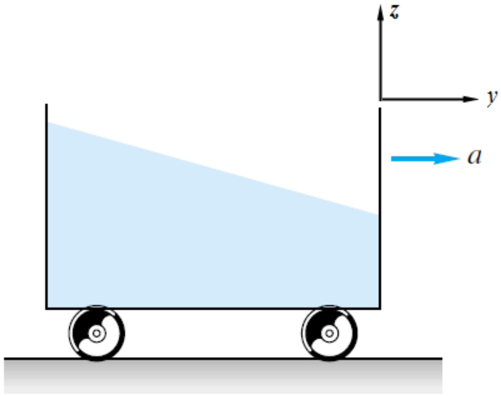
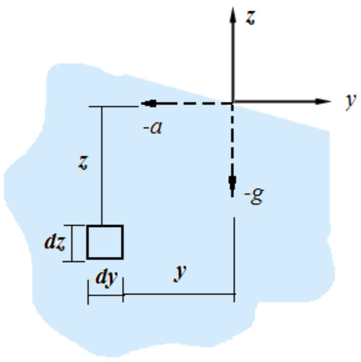
FLUID MECHANICS

LIQUIDS UNDER CONSTANT ACCELERATION OR UNDER CONSTANT ANGULAR SPEED

Fluids under Uniform Linear Acceleration

Consider a vehicle that speeding up in the y direction, containing liguid and open to the atmosphere above. At any point in the fluid, the force of gravity acts in the -z direction and the inertial force acts in the -y direction. A force does not occur in the perpendicular to the paper.





$P = \varrho\, g\, h$ is general pressure equations.

Let’s take smallest height (dz) and smallest length (dy). In this case pressures will occur are as follows.

$\int dP = \int \varrho(-\,g)\, dz$ $\qquad$ $\int dP = \int \varrho(-\,a)\, dy$

$P = -\varrho\, g\, z + C_1$ $\qquad$ $P = -\varrho\, a\, y + C_2$

$P = -\varrho\, g\, z - \varrho\, a\, y + (C_1 + C_2)$

Let’s take $(C_1 + C_2) = C_3$

$$\boxed{P = -\varrho\, g\, z - \varrho\, a\, y + C_3}$$

This equation gives the surfaces of equivalent pressure lines. Number of C varies according to Coordinate system. If we find the equation for the surface of fluid, it is as fallows.

In the surface P=0;

$P = -\varrho\, g\, z - \varrho\, a\, y + C_3$

$0 = -\varrho\, g\, z - \varrho\, a\, y + C_3$

If the origin on the surface of fluid and we find the pressure on the origin then z and y values will be zero. In this instance

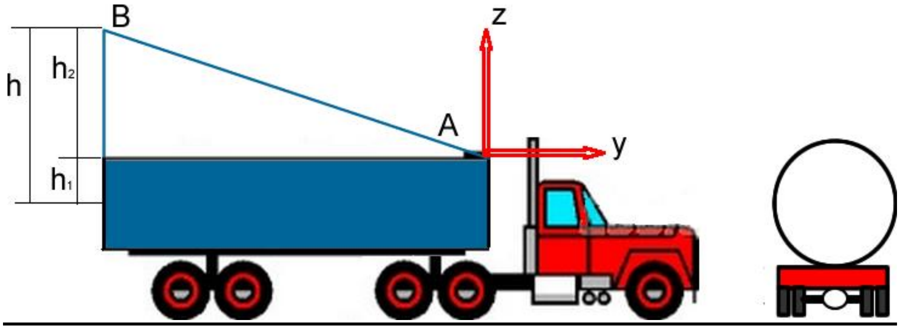
$0 = -0 - 0 + C_3 => C_3 = 0$

The relationship between y and z will be as follows.

$$\boxed{z = -\frac{a}{g} y}$$

This equation gives the function of the surface curve.

Example:As shown in the figure, the cylindrical tanker length L=8 m, diameter D=2.5 m. The tanker filled an oil with relative density ϱ=1.2. The A point of tanker is open to the atmosphere. Tanker speeding up by a=2 m/s^2 acceleration, find the pressure force and its center acting to back cover.



At first we must find the surface curve equation. If we put the coordinate system to point A, here the fluid pressure is 0. At this point, find the coefficient of C.

$P = -\varrho\, g\, z - \varrho\, a\, y + C$

$0 = 0 - 0 + C => C = 0$ y=0, z=0

In this case the equation is as follows.

$P = -\varrho\, g\, z - \varrho\, a\, y$ This is surface curve equation.

Using the values of y=8 m, a=2 m/s^2 , g= 9.81m/s^2, P_B =0

0 = -1200 kg/m^3 2 m/s^2 (-8 m)-1200 kg/m^3 9.81 m/s^2 h_2

h_2 = 1.63 m.

P_G =ϱ g h = 1200 kg/m^3 9.81 m/s^2 (1.63 + 1.25) m = 33903.36 N/m^2 (Pa).

The force to the cover; The pressure force on a plane surface immersed is equal to the center of gravity pressure and surface area multiplication.

F=P_G A = 33903.36 N/m^2 π 2.5^2 /4 m^2 = 166422 N ≈ 16 ton.

Let's find the center of pressure.

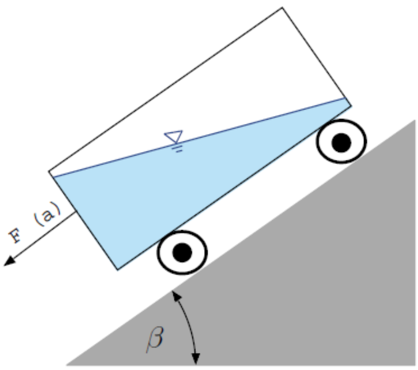
This force application point is not the G point where the average pressure. Below this point. We calculated in the previous lessons as follows.

$e = \frac{I_G}{h_G A}$ = (πD^4/64) / (h_G A) = (π 2.5^4/64) / [(1.63 + 1.25) (π 2.5^2/4)] =0.1356 m = 13.56 cm

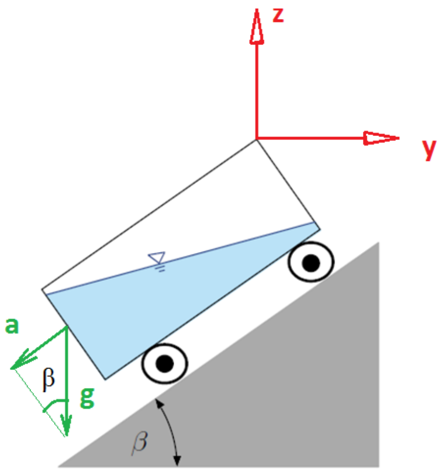
If the cover is in a vertical, $Z_G = H_G$

Example

A cart partially filled with liquid and is sliding on an inclined plane as shown in Figure. a) Calculate the shape of the surface. b) If there is a resistance, what will be the angle?



Solution



a) The angle can be found when the acceleration of the cart is found. If there is no resistance, the acceleration in the cart direction is determined from

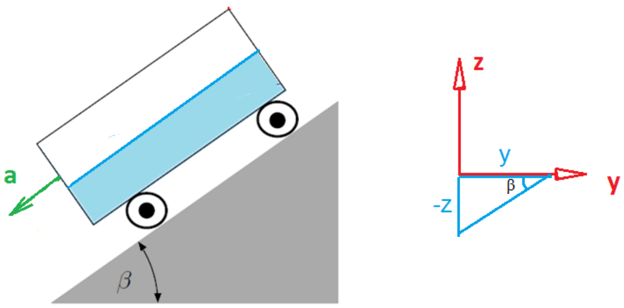
sinβ = a/g => a = g sinβ

The general pressure formula is
$P = -\varrho\, g\, z - \varrho\, a\, y + C$

On the surface this formula became like this;
$z = -\dfrac{a}{g}y$

'a' value is wroten their place;

$z = -\frac{\text{g sin}\beta}{g}y$ => sinβ = - z / y This is the surface equation. If ve draw this graphic, It is like this.

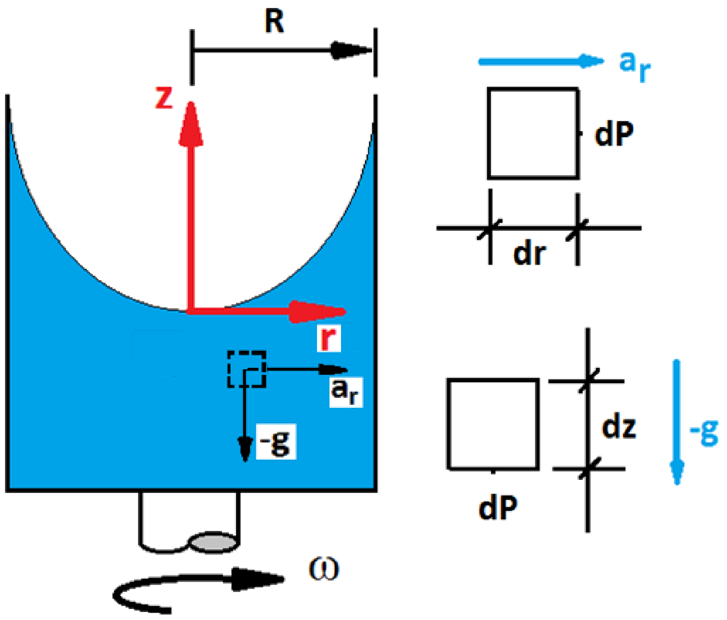


The effective body force is acting perpendicular to the slope. Thus, the liquid surface is parallel to the surface of the inclination surface.

b) In case of resistance force (either of friction due to the air or resistance in the wheels) reduces the acceleration of the cart.

Fluids under Constant Angular Speed

If fluid rotates around a fixed axis with a constant angular velocity then this motion is formed.



All accelaration has the their signs from the coordinate system. Accordingly the accelaration a_r is positive, that is centripetal acceleration and gravity accelaration is negatif. Radial pressure is effected from "a_r" accelaration and vertical pressure is effected from "–g" accelaration

Radial pressure;

$dP = \rho\, a_r\, dr$

$\int dP = \int \rho\, a_r\, dr$ $\qquad a_r = r\, \omega^2$

$\int P = \int \rho\, r\, \omega^2\, dr$

$P = \rho\, \omega^2 \int r\, dr$

$P = \rho\, \omega^2\, r^2 / 2 + C$ $\qquad$ This is radial direction pressure formulation

Vertical pressure;

$dP = \rho\, (-g)\, dz$

$dP = -\rho\, g\, dz$

$\int dP = \int -\rho\, g\, dz$

$P = -\rho\, g \int dz$

$P = -\rho\, g\, z + C$

Total pressure, due to both the radial and the vertical accelaration, is like this;

$\boxed{P = -\rho\, g\, z + \rho\, \omega^2\, r^2 / 2 + C}$

The pressure for the surface and location coordinate axis; At the surface the pressure is zero, at the origin the z and r coordinate values is zero. In this way;

$P = -\rho\, g\, z + \rho\, \omega^2\, r^2 / 2 + C$

$0 = -\rho\, g * 0 + \rho\, \omega^2 * 0 / 2 + C \Rightarrow C=0$ The formula is prufied

$-\rho\, g\, z + \rho\, \omega^2\, r^2 / 2 = 0$

$\rho\, g\, z = \rho\, \omega^2\, r^2 / 2$

$$z = \frac{\omega^2 r^2}{2g}$$

This is fluid surface equations at the same time.

Example