# FLUID MECHANICS

## MOVING FLUIDS

### Basic Concepts

**Incompressible Flow**: Liquids are much less compressible fluids. As long as there is very high pressures, the assumption can be that the density remains the same at every point in the fluid.

The flow is called as imcompressible flow that its density of fluid does not change and remaine the same at every point. The compressible flow is called when the density of fluid is changing.

**Steady Flow**: Steady flow is called that the flow variables (T, P, V, ρ) did not change with time. Otherwise, it is called non-steady flows.

After first starting the pump creates non-steady flow until it reaches regime. After reaching steady state creates a steady flow. If you change the pump speed, the state of steady flow will disrupt.

**Uniform Flow**: Average speed is the same every section, the flow take the name of uniform flow. Otherwise it is called non-uniform flows.

**(Flow Rate) Volume Flow**: In steady flow, It is called the specific volume of fluid that passing through a specific section in specific time. Indicated by $\dot{Q}$. Its unit is $m^3/s$. Its value is calculated by multiplying the cross-sectional area with the average speed of the fluid.

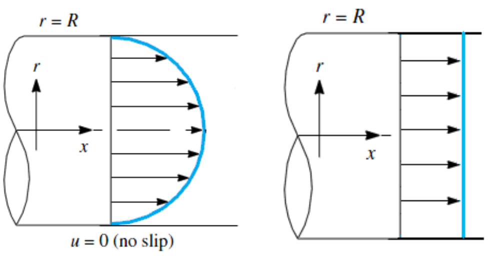
$\boxed{\dot{Q} = \bar{V} A}$

**Average Speed**: In a pipe, the speed reduce toward to margins and become zero at margin. Average Speed has uniform distribution in cross-sectional plane and perform real Flow rate. Therefore, the average speed used in calculations.

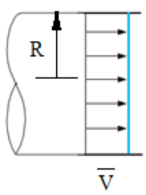
$\bar{V} = \dot{Q} / A$

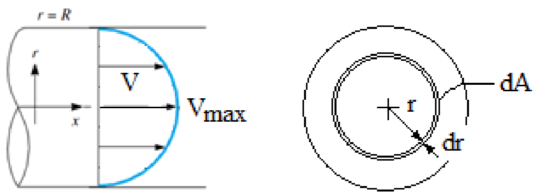
The velocity profile has parabolic shape in laminer circular flow. Average speed is half the maximum speed in parabolic flow.

$\boxed{\bar{V} = V_{max}/2}$




Proving the Average Speed: Let's prove that the average speed is half of the maximum speed.




$\dot{Q} = \bar{V} A$

$\dot{Q} = \bar{V} \pi R^2$

$dA = 2\pi r \, dr$

$A = \int_0^R 2\pi r \, dr$

$d\dot{Q} = V \, dA$

$d\dot{Q} = V \, 2\pi r \, dr$

$$\dot{Q}=\int_0^R V\,2\pi r\,dr$$

[Volume flow is calculated Average speed] = [Volume flow is calculated Parabolic flow]

$$\dot{Q} = \bar{V}\,\pi\,R^2 = \int_0^R V\,2\pi r\,dr$$

$$\bar{V} = \int_0^R \frac{V\,2\pi r\,dr}{\pi R2}$$

Here, V is a variable. It is need to be written as r form for integral calculation. Parabolic shape has got a second-order equation. So it is written as r form and a and b constant. (the second-order equation is like this $y=ax^2 + b$)

$V = a\,r^2 + b$ a and b may found boundary conditions

$V=V_{max}$ for $r=0 \Rightarrow V_{max} = 0 + b \Rightarrow b=V_{max}$

$V=0$ for $r=R \Rightarrow 0 = a\,R^2 + V_{max} \Rightarrow a= -V_{max}/R^2$

$V=[-V_{max}/R^2]\,r^2 + V_{max}$

Returning to average speed equation again

$$\bar{V}=\int_0^R \frac{\left[\frac{-Vmax}{R2}r2 + Vmax\right]2\pi r\,dr}{\pi R2}$$

.....

$$\bar{V}=\frac{2V_{max}}{R^2}\left(\int_0^R r dr - \frac{1}{R^2}\int_0^R r^3 dr\right)$$ C constant is zero for surface of fluid

.....

$$\bar{V} = \frac{2V_{max}}{R^2}\left(\frac{R^2}{4}\right)$$

$$\boxed{\bar{V} = \frac{V_{max}}{2}}$$

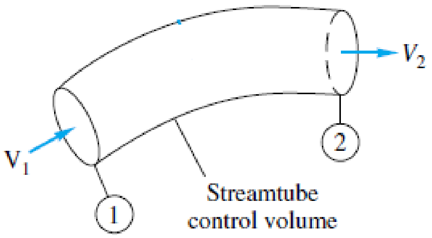
**Example**: 30 liters of water flows througy a pipe per second. Inner diameter is 200 mm. Calculate the average speed of the water.

$\dot{Q} = 30$ lt $= 30 /1000 = 0{,}030\ m^3 /s$

$A = \pi r^2 = 3{,}14 * 0{,}1^2\ m^2 = 0{,}0314\ m^2$

$\bar{V} = \dot{Q} / A = 0{,}030 / 0{,}0314 = 0{,}955$ m/s

**Conservation of Mass (Continuity Equation)**: Let us take a pipe in figure. If the fluid is imcompressible, the amount of entering mass have to equal to the amount of extracted mass.




[Inflow] = [Outflow]

$\varrho_1 \bar{V}_1 A_1 = \varrho_2 \bar{V}_2 A_2$ (if fluid is imcompressible $\Rightarrow \varrho_1 = \varrho_2$)

$\bar{V}_1 A_1 = \bar{V}_2 A_2$

In practicely, instead of the average speed usually mentioned as speed. Then the continuity equation is shown as follows.

$$\boxed{V_1A_1 = V_2A_2}$$

Don't forgot that the velocity here is average velocities.