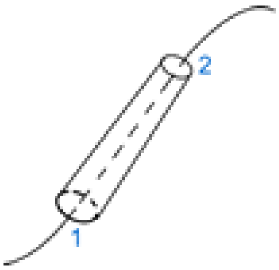
FLUID MECHANICS

THE BERNOULLI EQUATION (Frictionless Flow)



1 and 2 points are on the same flow line, between these two points, assuming that the specific energy of unit mass are equal, then we can write this equality.

$$g\,Z_1 + \frac{P_1}{\varrho} + \frac{V_1^2}{2} = g\,Z_2 + \frac{P_2}{\varrho} + \frac{V_2^2}{2}$$

This equation is valid in the following cases.

a) When the flow is steady flow and imcompressible flow.
b) In the absence of friction and losses
c) The force of gravity as the only external force is found
d) The two points are on the same flow line

Its unit is, $\frac{J}{kg} = \frac{N\,m}{kg} = \frac{m^2}{s}$

Let's explain the terms in Bernoulli equation.

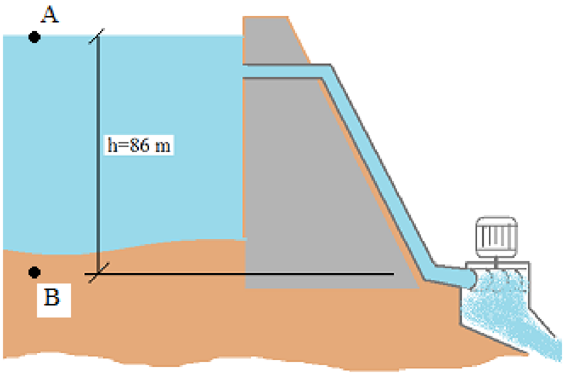
gZ Term: This expression indicates the Potential Energy of unit mass of fluid according to any reference plane.

P/ϱ Term: When the fluid moving in between 1 and 2 point, its indicates that the Work done by the force of pressure of unit mass of fluid. The displacement of the fluid is necassary to able work with the force of pressure.

$V^2/2$ Term: It is the Kinetic Energy of a unit mass of fluid.

According to Bernoulli equation, the specific energy of unit mass is remain constant in the same stream line along. Fluid velocity, pressure and height are transformed into each other. For the sum of these terms remain constant, each term may be changed. If a fluid lose height, It’s pressure or speed may increase.

Example:As shown in figure, the open surface of the dam is 86 m height from the turbines. a) Find the specific energy at A and B points. b)What would be the power plant, if this water energy can be converted into power with 250 m^3/s lossless flow rate.



a)

$E_A = g\,Z_A + \frac{P_A}{\varrho} + \frac{V_A^2}{2}$ $\{P_A = 0, V_A = 0\}$

$E_A = 9.81\frac{m}{s^2}\,86\,m + 0 + 0 = 843.66\frac{J}{kg}$

$E_B = g\,Z_B + \frac{P_B}{\varrho} + \frac{V_B^2}{2}$ $\{Z_B = 0, V_B = 0\}$

$E_B = 0 + \frac{\varrho\, g\, h}{\varrho} + 0 = 9.81 \frac{m}{s^2}\ 86\, m = 843.66 \frac{J}{kg}$

As can be seen here, specific energy is the same at every point of the water. Potential energy on surface shows itself in deep as pressure energy. For this process, one reference surface must be selected. This reference surface can be selected where wanted. The result will not change.

b) Every kg of water leave the specific energy to turbine. So we can find the power turbine by multiplying the specific eneryg with mass flow rate.

[Power] = [mass flow rate] * [specific enerji]

$\boxed{P = \dot{m}\, E}$

[mass flow rate] = [density] [volumetric flow rate]

$\boxed{\dot{m} = \rho\, \dot{Q}}$

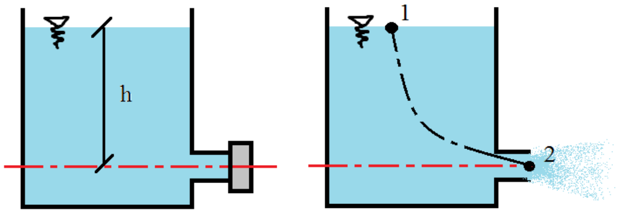
F = m a
N = kg m/s^2
kg = N s^2 /m

P = [ϱ $\dot{Q}$] [g h]

= [1000 kg/m^3 250 m^3/s] [9.81 m/s^2 86 m]

= 210915000 {kg m^2/s^3 = Nm/s=J/s=W} = 210915 kW

Reservoir Outlets: Get off the mouth of a container as shown in the figure. In this case the potential energy is found at point 1 and presssure energy is found at point 2. When the mouth of the container is opened, only kinetic energy is found at point 2. Because effective pressure of every open surface to air is zero.



When the plug is removed, the water flows with velocity V as beam. Effective pressure is zero at the output. Because of the speed has only kinetic energy.

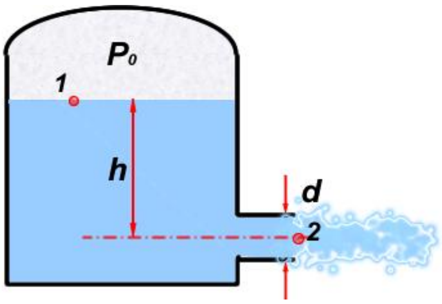
$g\, Z_1 + \frac{P_1}{\varrho} + \frac{V_1^2}{2} = g\, Z_2 + \frac{P_2}{\varrho} + \frac{V_2^2}{2}$ { Z_1 =h, P_1=0, V_1=0 // Z_2=0, P_2=0, V_2=?}

gh + 0 + 0 = 0 + 0 + $V_2^2/2$

$V_2 = \sqrt{2\, g\, h}$

This value is smaller than the output speed due to friction losses.

<u>**Example**</u>: The air pressure is P_0=1.65 bar at the top of the container with filled water a certain level. Find the speed, flow rate and power at the open jet (d= 5 cm, h=2,2 m).



Let's write the Bernoulli equation between 1 and 2 points.

$g\, Z_1 + \frac{P_1}{\varrho} + \frac{V_1^2}{2} = g\, Z_2 + \frac{P_2}{\varrho} + \frac{V_2^2}{2}$ { Z_1 =h=2.2m, P_1=1,65 bar, V_1=0 // Z_2=0, P_2=0, V_2=?}

9.81*2.2 + 165000/1000 + 0 = 0 + 0 + $V_2^2/2$

$V_2 = \sqrt{2(9.81 * 2.2 + \frac{165000}{1000})}$

V_2 = 19.32 m/s

For volumetric flow rate;

$\dot{Q} = \bar{V}\ A$

$\dot{Q}$ = 19.32 m/s * (3.14 * 0.05^2 /4) = 0.0379 m^3 /s

Q = 37.9 liter/s

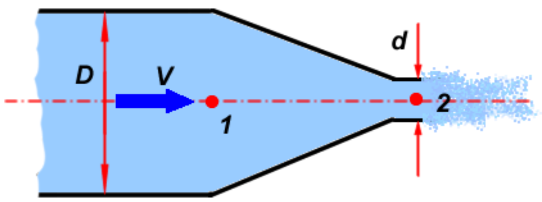
For power;

$P = \ \dot{m}\, E = \rho\, \dot{Q}\, E = \rho\, \dot{Q}\, \frac{V_2^2}{2}$ $\{ \dot{m} = \ \rho\, \dot{Q} \quad //\ E = E_2 = g\, Z_2 + \frac{P_2}{\varrho} + \frac{V_2^2}{2} = 0 + 0 + \frac{V_2^2}{2} \}$

P = 1000 kg/m^3 *0.0379 m^3/s * 19.32^2 /2

P = 7073 Nm/s [Watt] ≈ 7 kW

Example:Find the pressure in inlet cross section of sprayer as shown in the figure (D=10 cm, d=2.5 cm, V=2.2m/s).



From Continuity Equation (inlet flow rate = output flow rate)

$\dot{Q}_1 = \dot{Q}_1$

$V_1A_1 = V_2A_2$

2.2 m/s* (3.14*0.1^2/4) m^2 = V2* (3.14*0.025^2/4)m^2

V_2=35.2 m/s

Let's write the Bernoulli equation between 1 and 2 points.

$g\, Z_1 + \frac{P_1}{\varrho} + \frac{V_1^2}{2} = \ g\, Z_2 + \frac{P_2}{\varrho} + \frac{V_2^2}{2}$ { Z_1 = 0, P_1=?, V_1=2.2 m/s // Z_2=0, P_2=0, V_2=35.2}

0 + P_1/1000 + 2.2^2 /2 = 0 + 0 + 35.2^2 /2

P_1 = 617100 Pa [N/m^2] =6.17 bar