

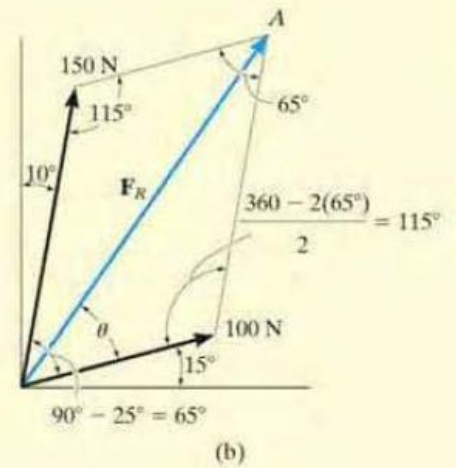
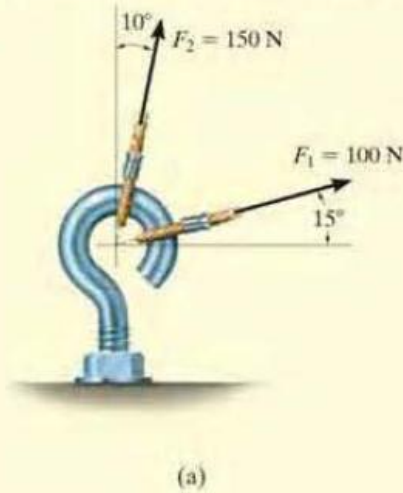
FİNAL VE BÜTÜNLEME SINAVLARINA HAZIRLIK ÇALIŞMA SORULARI

Aşağıdaki sorulara bire bir çalışmayın. Sorunun temel mantığı nedir ona göre çalışın. Soruların detayı incelenmedi. Sadece şekline bakarak karar verildi. Temel olarak bileşke vektörün ve bileşenlerin bulunması ile ilgili soru gelecektir. Şekil olarak buradaki soruların şekli aynen kullanılacak.

BİLEŞKE KUVVET ve BİLEŞENLERİN BULUNMASI

Örnek

The screw eye in Fig. 2-11a is subjected to two forces, \mathbf{F}_1 and \mathbf{F}_2 . Determine the magnitude and direction of the resultant force.



SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of \mathbf{F}_1 that is parallel to \mathbf{F}_2 , and another line from the head of \mathbf{F}_2 that is parallel to \mathbf{F}_1 . The resultant force \mathbf{F}_R extends to where these lines intersect at point A, Fig. 2-11b. The two unknowns are the magnitude of \mathbf{F}_R and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2-11c. Using the law of cosines

$$\begin{aligned} F_R &= \sqrt{(100 \text{ N})^2 + (150 \text{ N})^2 - 2(100 \text{ N})(150 \text{ N}) \cos 115^\circ} \\ &= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6 \text{ N} \\ &= 213 \text{ N} \end{aligned}$$

Ans.

Applying the law of sines to determine θ ,

$$\begin{aligned} \frac{150 \text{ N}}{\sin \theta} &= \frac{212.6 \text{ N}}{\sin 115^\circ} & \sin \theta &= \frac{150 \text{ N}}{212.6 \text{ N}} (\sin 115^\circ) \\ \theta &= 39.8^\circ \end{aligned}$$

Thus, the direction ϕ (phi) of \mathbf{F}_R , measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ$$

Ans.

NOTE: The results seem reasonable, since Fig. 2-11b shows \mathbf{F}_R to have a magnitude larger than its components and a direction that is between them.

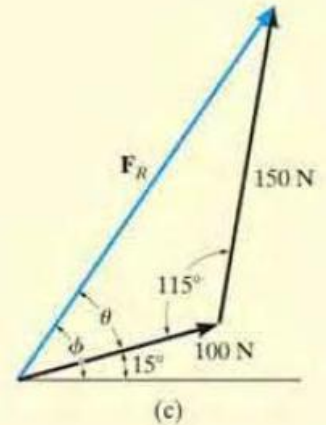
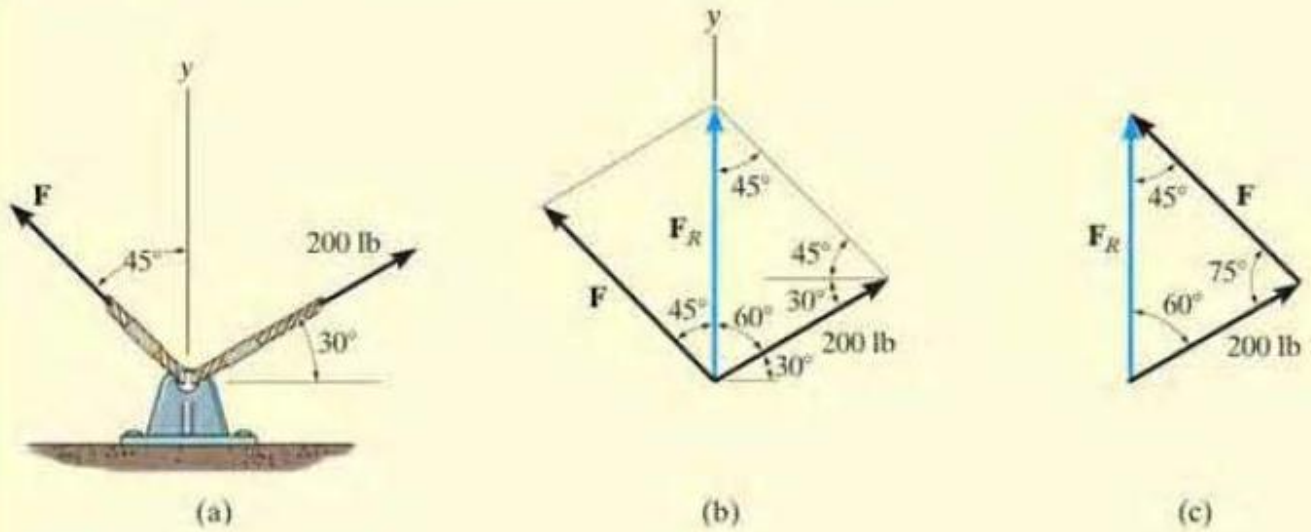


Fig. 2-11

Örnek

Determine the magnitude of the component force \mathbf{F} in Fig. 2–13a and the magnitude of the resultant force \mathbf{F}_R if \mathbf{F}_R is directed along the positive y axis.



SOLUTION

The parallelogram law of addition is shown in Fig. 2–13b, and the triangle rule is shown in Fig. 2–13c. The magnitudes of \mathbf{F}_R and \mathbf{F} are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

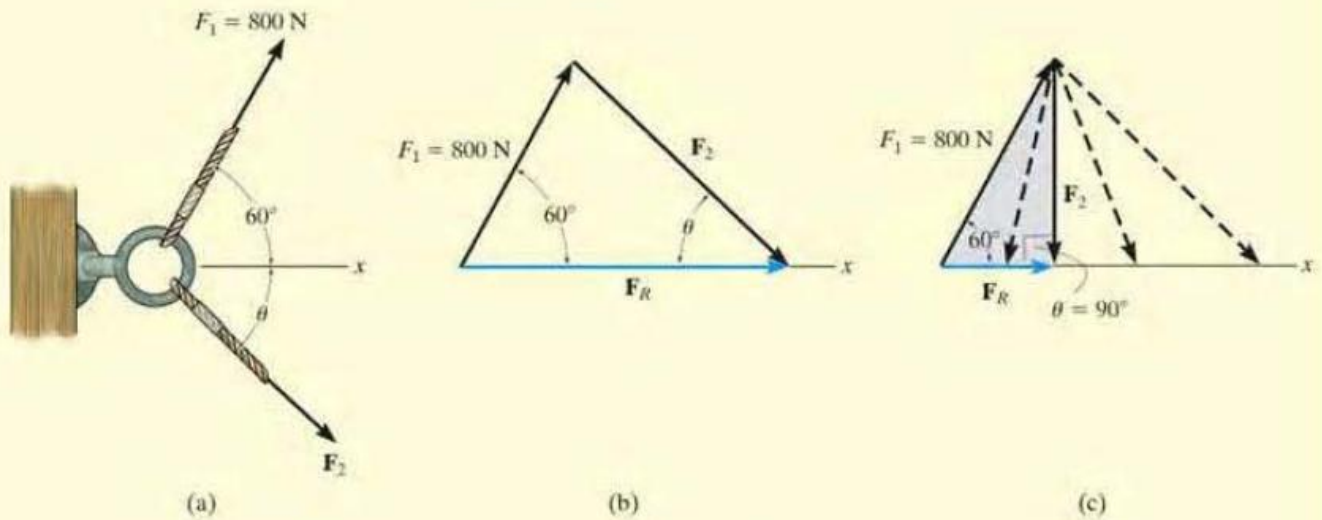
$$F = 245\text{ lb} \quad \text{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

$$F_R = 273\text{ lb} \quad \text{Ans.}$$

Örnek

It is required that the resultant force acting on the eyebolt in Fig. 2-14a be directed along the positive x axis and that \mathbf{F}_2 have a *minimum* magnitude. Determine this magnitude, the angle θ , and the corresponding resultant force.



SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2-14b. Since the magnitudes (lengths) of \mathbf{F}_R and \mathbf{F}_2 are not specified, then \mathbf{F}_2 can actually be any vector that has its head touching the line of action of \mathbf{F}_R , Fig. 2-14c. However, as shown, the magnitude of \mathbf{F}_2 is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of \mathbf{F}_R , that is, when

$$\theta = 90^\circ \quad \text{Ans.}$$

Since the vector addition now forms a right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800 \text{ N})\cos 60^\circ = 400 \text{ N} \quad \text{Ans.}$$

$$F_2 = (800 \text{ N})\sin 60^\circ = 693 \text{ N} \quad \text{Ans.}$$

Örnek

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction measured counterclockwise from the positive u axis.

Given:

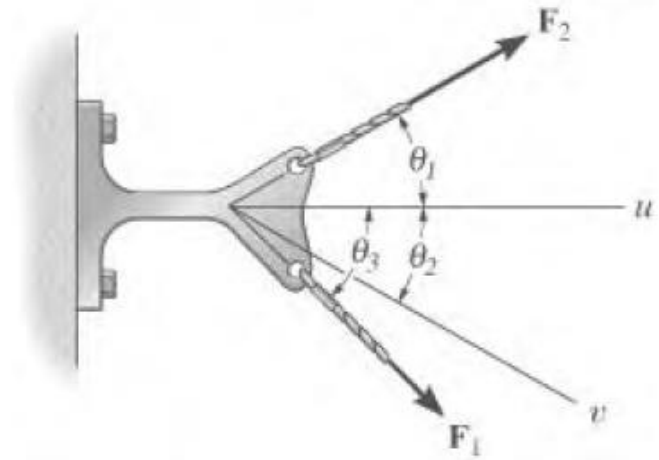
$$F_1 = 25 \text{ lb}$$

$$F_2 = 50 \text{ lb}$$

$$\theta_1 = 30^\circ$$

$$\theta_2 = 30^\circ$$

$$\theta_3 = 45^\circ$$



Solution:

$$\alpha = 180^\circ - (\theta_3 + \theta_1)$$

$$F_R = \sqrt{F_2^2 + F_1^2 - 2 F_1 F_2 \cos(\alpha)}$$

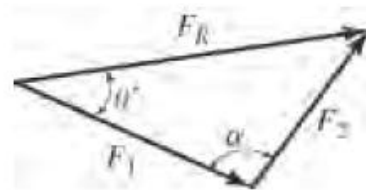
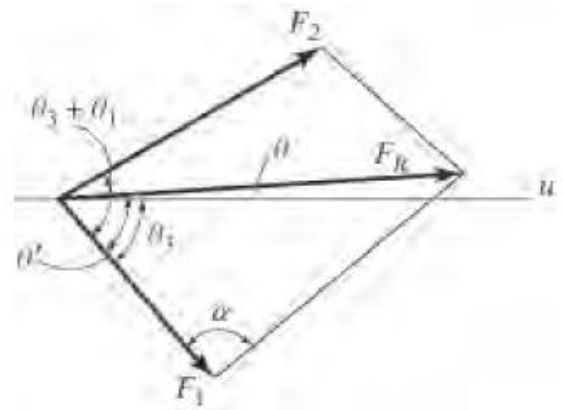
$$F_R = 61.4 \text{ lb}$$

$$\frac{\sin(\theta)}{F_2} = \frac{\sin(\alpha)}{F_R} \quad \theta' = \arcsin\left(\sin(\alpha) \frac{F_2}{F_R}\right)$$

$$\theta' = 51.8^\circ$$

$$\theta = \theta' - \theta_3$$

$$\theta = 6.8^\circ$$



Örnek

A resultant force \mathbf{F} is necessary to hold the balloon in place. Resolve this force into components along the tether lines AB and AC , and compute the magnitude of each component.

Given:

$$F = 350 \text{ lb}$$

$$\theta_1 = 30^\circ$$

$$\theta_2 = 40^\circ$$

Solution:

$$\frac{F_{AB}}{\sin(\theta_1)} = \frac{F}{\sin[180^\circ - (\theta_1 + \theta_2)]}$$

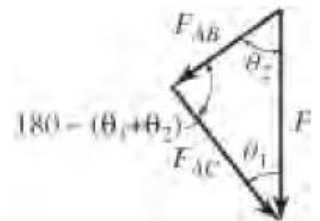
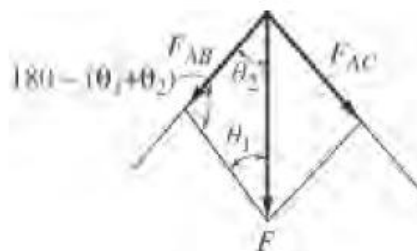
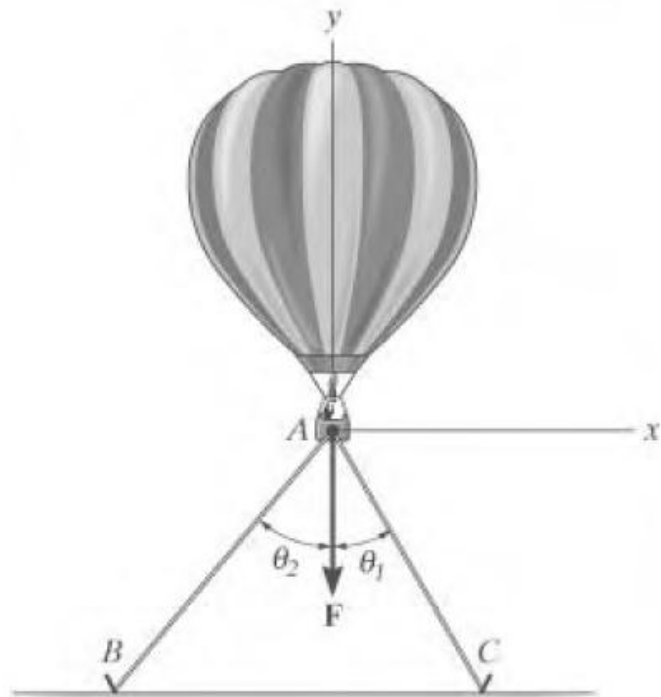
$$F_{AB} = F \left[\frac{\sin(\theta_1)}{\sin[180^\circ - (\theta_1 + \theta_2)]} \right]$$

$$F_{AB} = 186 \text{ lb}$$

$$\frac{F_{AC}}{\sin(\theta_2)} = \frac{F}{\sin[180^\circ - (\theta_1 + \theta_2)]}$$

$$F_{AC} = F \left[\frac{\sin(\theta_2)}{\sin[180^\circ - (\theta_1 + \theta_2)]} \right]$$

$$F_{AC} = 239 \text{ lb}$$



Örnek

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

Given:

$$F_1 = 60 \text{ lb}$$

$$F_2 = 70 \text{ lb}$$

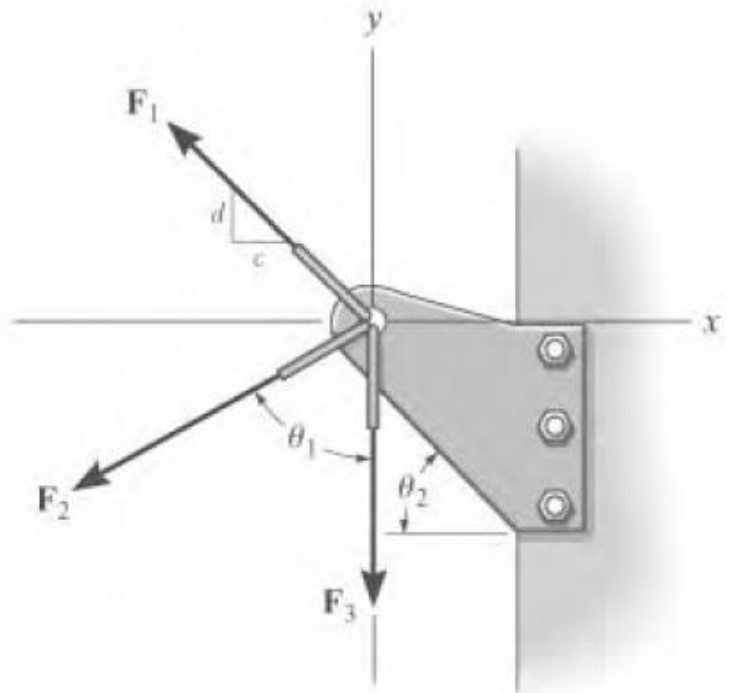
$$F_3 = 50 \text{ lb}$$

$$\theta_1 = 60^\circ$$

$$\theta_2 = 45^\circ$$

$$c = 1$$

$$d = 1$$



Solution:

Örnek

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket.

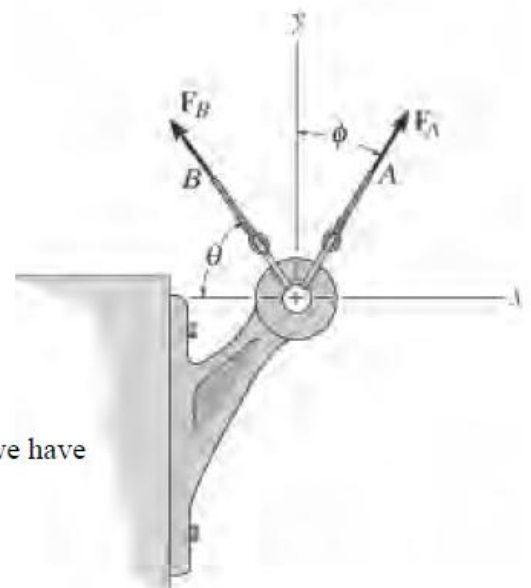
Given:

$$F_A = 700 \text{ N}$$

$$F_B = 600 \text{ N}$$

$$\theta = 20^\circ$$

$$\phi = 30^\circ$$



Solution:

Scalar Notation: Summing the force components algebraically, we have

$$F_{Rx} = \sum F_x; \quad F_{Rx} = F_A \sin(\phi) - F_B \cos(\theta)$$

$$F_{Rx} = -213.8 \text{ N}$$

$$F_{Ry} = \Sigma F_y; \quad F_{Ry} = F_A \cos(\phi) + F_B \sin(\theta)$$

$$F_{Ry} = 811.4 \text{ N}$$

The magnitude of the resultant force F_R is

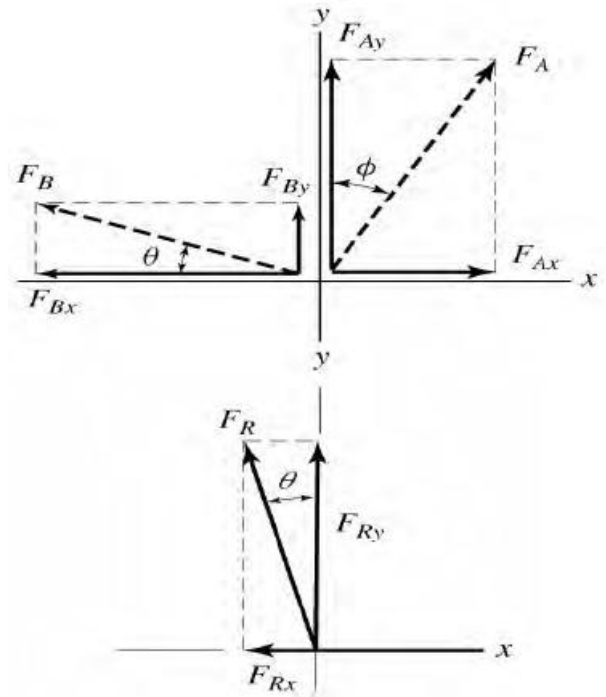
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$F_R = 839 \text{ N}$$

The directional angle θ measured counterclockwise from the positive x axis is

$$\theta = \text{atan}\left(\frac{|F_{Rx}|}{F_{Ry}}\right)$$

$$\theta = 14.8 \text{ deg}$$



Örnek

Determine the x and y components of each force acting on the *gusset plate* of the bridge truss.

Given:

$$F_1 = 200 \text{ lb} \quad c = 3$$

$$F_2 = 400 \text{ lb} \quad d = 4$$

$$F_3 = 300 \text{ lb} \quad e = 3$$

$$F_4 = 300 \text{ lb} \quad f = 4$$

Solution:

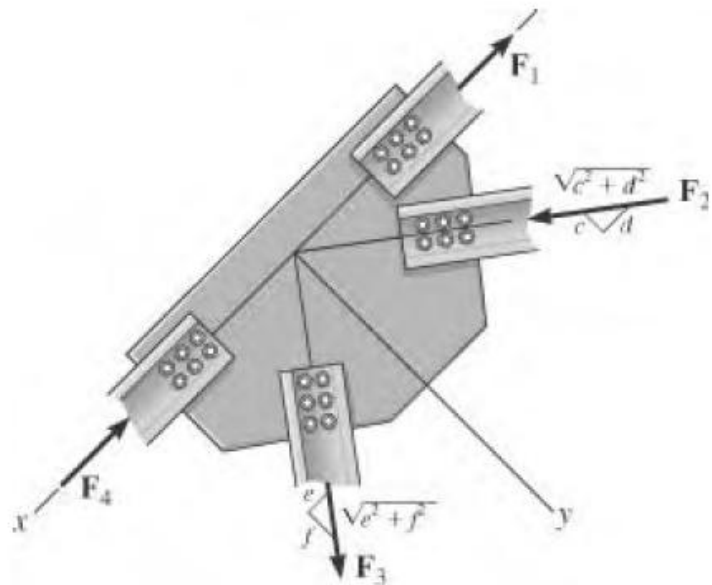
$$F_{1x} = -F_1$$

$$F_{1x} = -200 \text{ lb}$$

$$F_{1y} = 0 \text{ lb}$$

$$F_{2x} = F_2 \left(\frac{d}{\sqrt{c^2 + d^2}} \right)$$

$$F_{2x} = 320 \text{ lb}$$



$$F_{2y} = -F_2 \left(\frac{c}{\sqrt{c^2 + d^2}} \right)$$

$$F_{2y} = -240 \text{ lb}$$

$$F_{3x} = F_3 \left(\frac{e}{\sqrt{e^2 + f^2}} \right)$$

Örnek

Determine the magnitude of force **F** so that the resultant **F_R** of the three forces is as small as possible. What is the minimum magnitude of **F_R**?

Units Used:

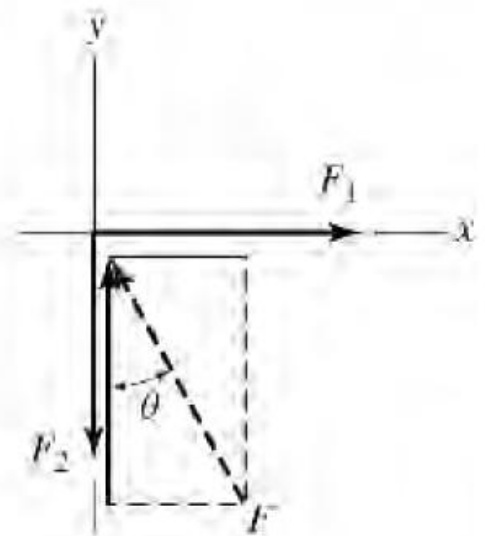
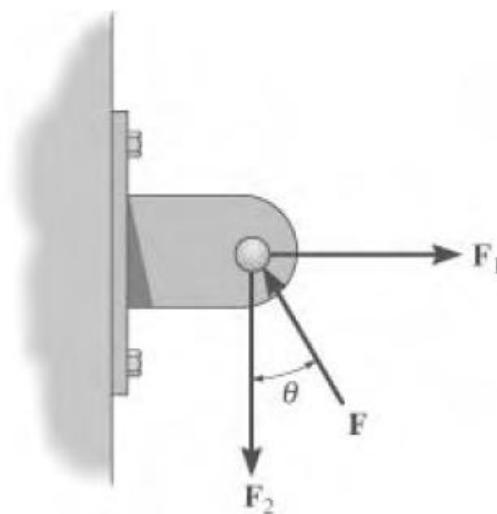
$$\text{kN} = 1000 \text{ N}$$

Given:

$$F_1 = 5 \text{ kN}$$

$$F_2 = 4 \text{ kN}$$

$$\theta = 30^\circ$$

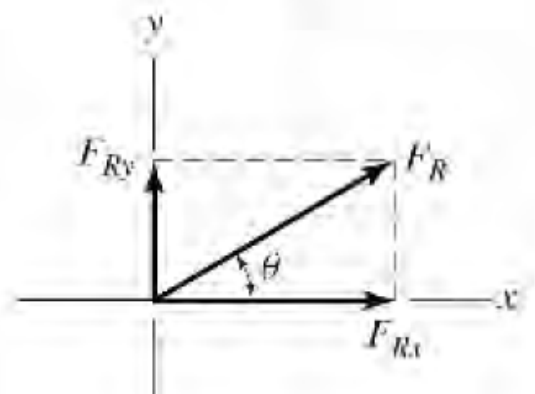


Solution:

Scalar Notation: Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow \quad F_{Rx} &= \sum F_x; \quad F_{Rx} = F_1 - F \sin(\theta) \end{aligned}$$

$$\begin{aligned} \uparrow \quad F_{Ry} &= \sum F_y; \quad F_{Ry} = F \cos(\theta) - F_2 \end{aligned}$$



The magnitude of the resultant force **F_R** is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(F_1 - F \sin(\theta))^2 + (F \cos(\theta) - F_2)^2}$$

$$F_R^2 = F_1^2 + F_2^2 + F^2 - 2FF_1 \sin(\theta) - 2F_2 F \cos(\theta)$$

$$2F_R \frac{dF_R}{dF} = 2F - 2F_1 \sin(\theta) - 2F_2 \cos(\theta)$$

If F is a minimum, then $\left(\frac{dF_R}{dF} = 0 \right) \quad F = F_1 \sin(\theta) + F_2 \cos(\theta) \quad F = 5.96 \text{ kN}$

$$F_R = \sqrt{(F_1 - F \sin(\theta))^2 + (F \cos(\theta) - F_2)^2} \quad F_R = 2.3 \text{ kN}$$

Örnek

Three forces act on the bracket. Determine the magnitude and orientation θ of \mathbf{F}_2 so that the resultant force is directed along the positive u axis and has magnitude F_R .

Given:

$$F_R = 50 \text{ lb}$$

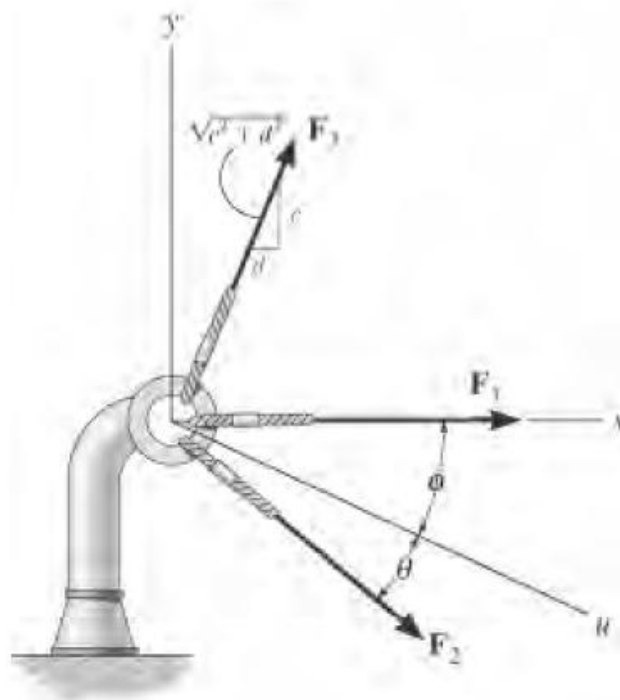
$$F_1 = 80 \text{ lb}$$

$$F_3 = 52 \text{ lb}$$

$$\phi = 25^\circ$$

$$c = 12$$

$$d = 5$$



Solution:

Guesses

$$F_2 = 1 \text{ lb} \quad \theta = 120 \text{ deg}$$

Given

$$F_R \cos(\phi) = F_1 + F_2 \cos(\phi + \theta) + \left(\frac{d}{\sqrt{c^2 + d^2}} \right) F_3$$

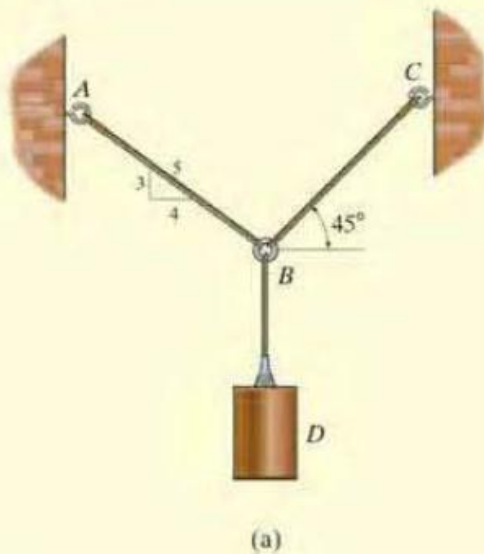
$$-F_R \sin(\phi) = -F_2 \sin(\phi + \theta) + \left(\frac{c}{\sqrt{c^2 + d^2}} \right) F_3$$

$$\begin{pmatrix} F_2 \\ \theta \end{pmatrix} = \text{Find}(F_2, \theta) \quad F_2 = 88.1 \text{ lb} \quad \theta = 103.3 \text{ deg}$$

DENGE

Örnek

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3-6a.



SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be $T_{BD} = 60(9.81)$ N, Fig. 3-6*b*. The forces in cables BA and BC can be determined by investigating the equilibrium of ring B . Its free-body diagram is shown in Fig. 3-6*c*. The magnitudes of T_A and T_C are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\rightarrow \Sigma F_x = 0; \quad T_C \cos 45^\circ - \left(\frac{4}{5}\right)T_A = 0 \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\frac{3}{5}\right)T_A - 60(9.81) \text{ N} = 0 \quad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\frac{3}{5}\right)(0.8839T_C) - 60(9.81) \text{ N} = 0$$

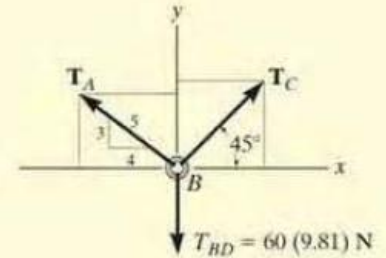
So that

$$T_C = 475.66 \text{ N} = 476 \text{ N} \quad \text{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420 \text{ N} \quad \text{Ans.}$$

NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.

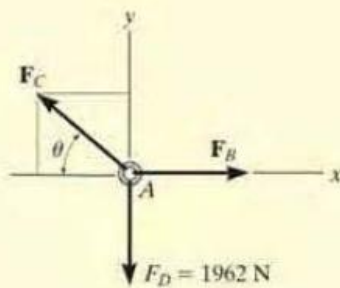


(c)

Fig. 3-6

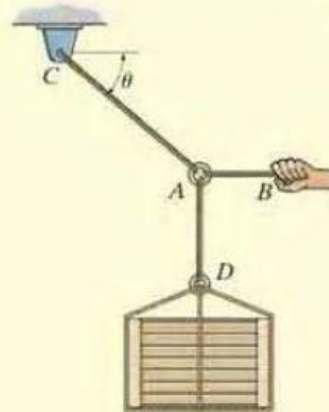
Örnek

The 200-kg crate in Fig. 3-7*a* is suspended using the ropes AB and AC . Each rope can withstand a maximum force of 10 kN before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.



(b)

Fig. 3-7



(a)

SOLUTION

Free-Body Diagram. We will study the equilibrium of ring A . There are three forces acting on it, Fig. 3-7b. The magnitude of F_D is equal to the weight of the crate, i.e., $F_D = 200(9.81) \text{ N} = 1962 \text{ N} < 10 \text{ kN}$.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes,

$$\rightarrow \Sigma F_x = 0; \quad -F_C \cos \theta + F_B = 0; \quad F_C = \frac{F_B}{\cos \theta} \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad F_C \sin \theta - 1962 \text{ N} = 0 \quad (2)$$

From Eq. (1), F_C is always greater than F_B since $\cos \theta \leq 1$. Therefore, rope AC will reach the maximum tensile force of 10 kN before rope AB . Substituting $F_C = 10 \text{ kN}$ into Eq. (2), we get

$$[10(10^3) \text{ N}] \sin \theta - 1962 \text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ \quad \text{Ans.}$$

The force developed in rope AB can be obtained by substituting the values for θ and F_C into Eq. (1).

$$10(10^3) \text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

$$F_B = 9.81 \text{ kN}$$

Örnek

Determine the required length of cord AC in Fig. 3-8a so that the 8-kg lamp can be suspended in the position shown. The undeformed length of spring AB is $l'_{AB} = 0.4 \text{ m}$, and the spring has a stiffness of $k_{AB} = 300 \text{ N/m}$.

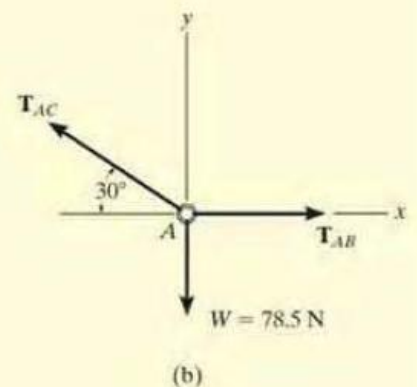
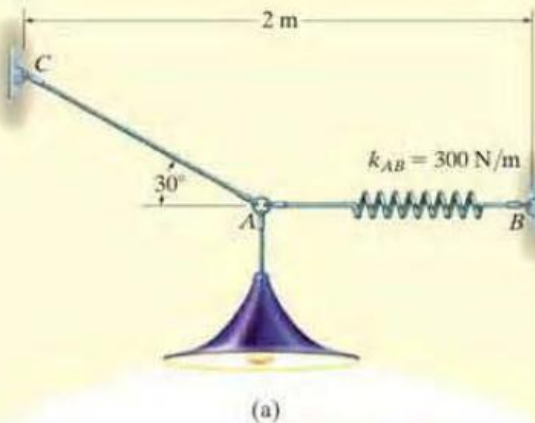


Fig. 3-8

SOLUTION

If the force in spring AB is known, the stretch of the spring can be found using $F = ks$. From the problem geometry, it is then possible to calculate the required length of AC .

Free-Body Diagram. The lamp has a weight $W = 8(9.81) = 78.5 \text{ N}$ and so the free-body diagram of the ring at A is shown in Fig. 3–8b.

Equations of Equilibrium. Using the x, y axes,

$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & T_{AB} - T_{AC} \cos 30^\circ &= 0 \\ +\uparrow \Sigma F_y &= 0; & T_{AC} \sin 30^\circ - 78.5 \text{ N} &= 0 \end{aligned}$$

Solving, we obtain

$$T_{AC} = 157.0 \text{ N}$$

$$T_{AB} = 135.9 \text{ N}$$

The stretch of spring AB is therefore

$$T_{AB} = k_{AB}s_{AB}; \quad 135.9 \text{ N} = 300 \text{ N/m}(s_{AB})$$

$$s_{AB} = 0.453 \text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4 \text{ m} + 0.453 \text{ m} = 0.853 \text{ m}$$

The horizontal distance from C to B , Fig. 3–8a, requires

$$2 \text{ m} = l_{AC} \cos 30^\circ + 0.853 \text{ m}$$

$$l_{AC} = 1.32 \text{ m}$$

Ans.

Örnek

The crate of weight W is hoisted using the ropes AB and AC . Each rope can withstand a maximum tension T before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted.

Given:

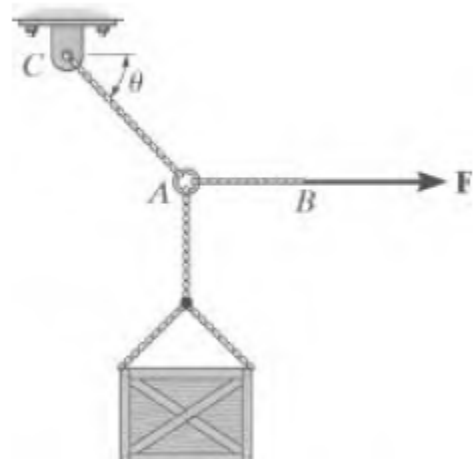
$$W = 500 \text{ lb}$$

$$T = 2500 \text{ lb}$$

Solution:

Case 1: Assume $T_{AB} = T$

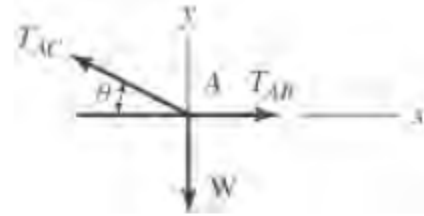
$$\text{The initial guess} \quad \theta = 30^\circ \quad T_{AC} = 2000 \text{ lb}$$



Given

$$\rightarrow \Sigma F_x = 0; \quad T_{AB} - T_{AC} \cos(\theta) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T_{AC} \sin(\theta) - W = 0$$



$$\begin{pmatrix} T_{AC1} \\ \theta_1 \end{pmatrix} = \text{Find}(T_{AC}, \theta) \quad \theta_1 = 11.31 \text{ deg} \quad T_{AC1} = 2550 \text{ lb}$$

Case 1: Assume $T_{AC} = T$

The initial guess $\theta = 30 \text{ deg}$ $T_{AB} = 2000 \text{ lb}$

Given

$$\rightarrow \Sigma F_x = 0; \quad T_{AB} - T_{AC} \cos(\theta) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T_{AC} \sin(\theta) - W = 0$$

$$\begin{pmatrix} T_{AB2} \\ \theta_2 \end{pmatrix} = \text{Find}(T_{AB}, \theta) \quad \theta_2 = 11.54 \text{ deg} \quad T_{AB2} = 2449 \text{ lb}$$

$$\theta = \max(\theta_1, \theta_2) \quad \theta = 11.54 \text{ deg}$$

Örnek

Determine the force in each cable and the force **F** needed to hold the lamp of mass *M* in the position shown. *Hint:* First analyze the equilibrium at *B*; then, using the result for the force in *BC*, analyze the equilibrium at *C*.

Given:

$$M = 4 \text{ kg}$$

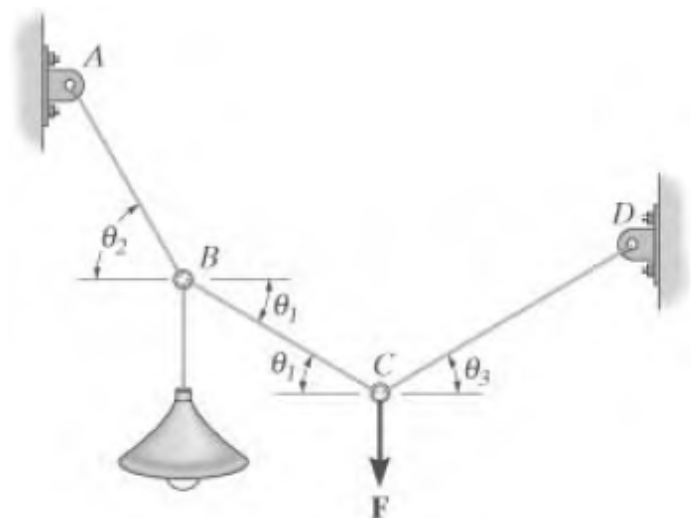
$$\theta_1 = 30 \text{ deg}$$

$$\theta_2 = 60 \text{ deg}$$

$$\theta_3 = 30 \text{ deg}$$

Solution:

Initial guesses:



$$T_{BC} = 1 \text{ N} \quad T_{BA} = 2 \text{ N}$$

Given

At B:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T_{BC} \cos(\theta_1) - T_{BA} \cos(\theta_2) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T_{BA} \sin(\theta_2) - T_{BC} \sin(\theta_1) - Mg = 0$$

$$\begin{pmatrix} T_{BC} \\ T_{BA} \end{pmatrix} = \text{Find}(T_{BC}, T_{BA}) \quad \begin{pmatrix} T_{BC} \\ T_{BA} \end{pmatrix} = \begin{pmatrix} 39.24 \\ 67.97 \end{pmatrix} \text{ N}$$

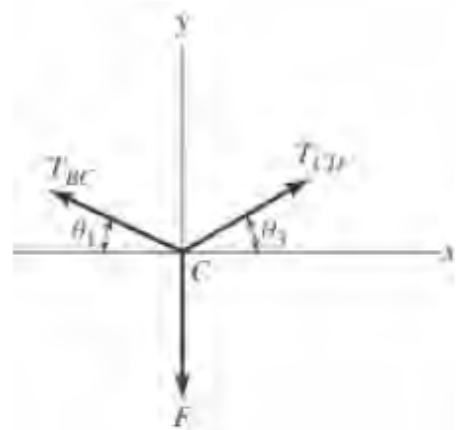
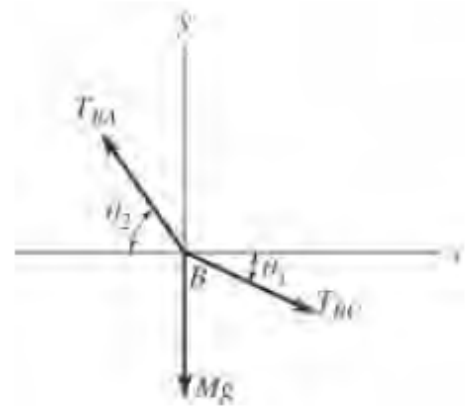
$$\text{At C:} \quad T_{CD} = 1 \text{ N} \quad F = 2 \text{ N}$$

Given

$$\xrightarrow{+} \Sigma F_x = 0; \quad -T_{BC} \cos(\theta_1) + T_{CD} \cos(\theta_3) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T_{BC} \sin(\theta_1) + T_{CD} \sin(\theta_3) - F = 0$$

$$\begin{pmatrix} T_{CD} \\ F \end{pmatrix} = \text{Find}(T_{CD}, F) \quad \begin{pmatrix} T_{CD} \\ F \end{pmatrix} = \begin{pmatrix} 39.24 \\ 39.24 \end{pmatrix} \text{ N}$$



REAKSİYON KUVVETLERİ VE DENGİ

Örnek

Draw the free-body diagram of the jib crane AB , which is pin-connected at A and supported by member (link) BC .

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$F = 8 \text{ kN}$$

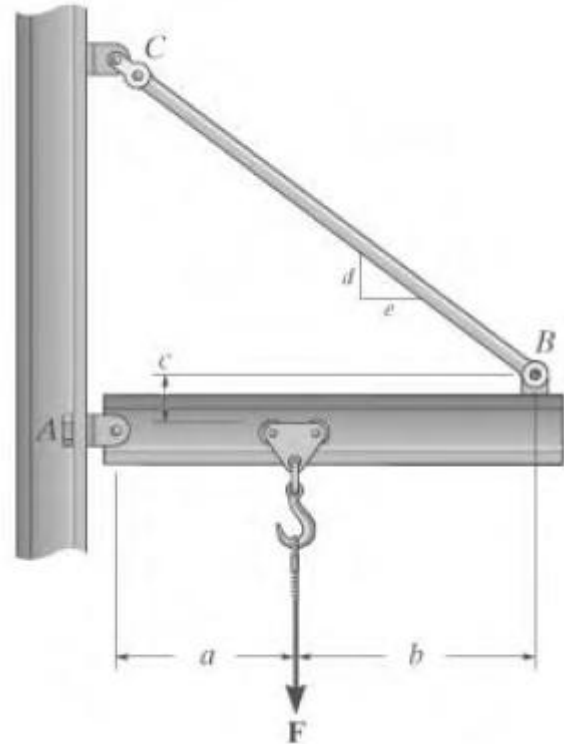
$$a = 3 \text{ m}$$

$$b = 4 \text{ m}$$

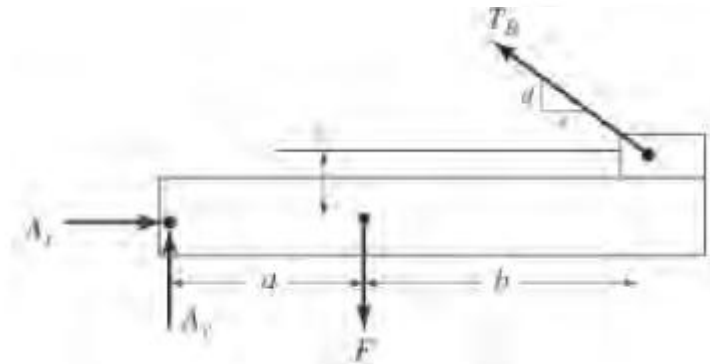
$$c = 0.4 \text{ m}$$

$$d = 3$$

$$e = 4$$



Solution:



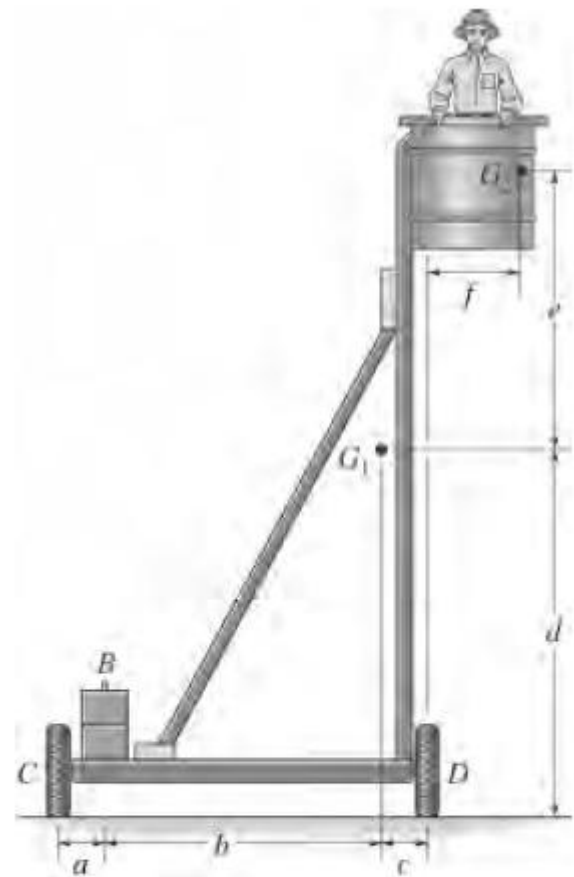
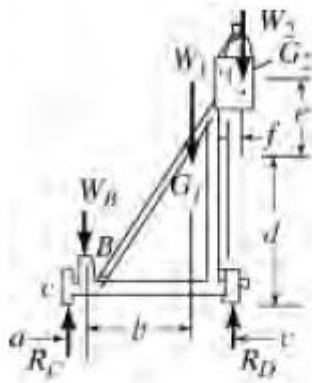
Örnek

The platform assembly has weight W_1 and center of gravity at G_1 . If it is intended to support a maximum load W_2 placed at point G_2 , determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.

Given:

$$W_1 = 250 \text{ lb} \quad a = 1 \text{ ft} \quad c = 1 \text{ ft} \quad e = 6 \text{ ft}$$

$$W_2 = 400 \text{ lb} \quad b = 6 \text{ ft} \quad d = 8 \text{ ft} \quad f = 2 \text{ ft}$$



Solution:

When tipping occurs, $R_c = 0$

$$\curvearrowleft \Sigma M_D = 0; \quad -W_2 f + W_1 c + W_B (b + c) = 0$$

$$W_B = \frac{W_2 f - W_1 c}{b + c}$$

$$W_B = 78.6 \text{ lb}$$

Örnek

The articulated crane boom has a weight W and mass center at G . If it supports a load L , determine the force acting at the pin A and the compression in the hydraulic cylinder BC when the boom is in the position shown.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$W = 125 \text{ lb}$$

$$L = 600 \text{ lb}$$

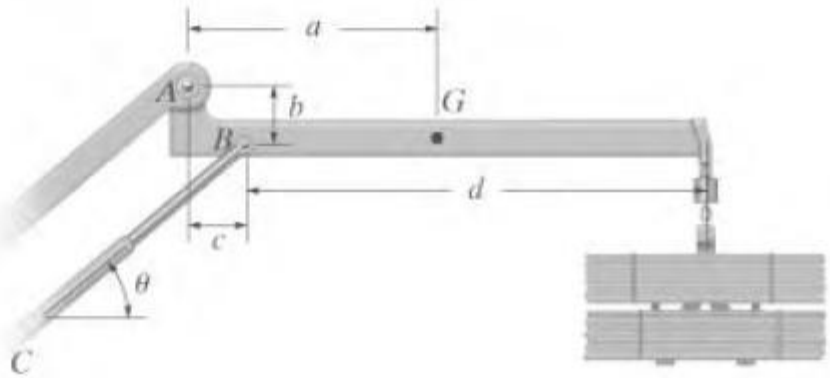
$$a = 4 \text{ ft}$$

$$b = 1 \text{ ft}$$

$$c = 1 \text{ ft}$$

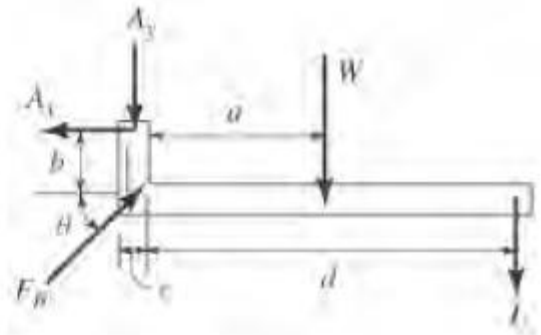
$$d = 8 \text{ ft}$$

$$\theta = 40^\circ$$



Solution:

Guesses $A_x = 1 \text{ lb}$ $A_y = 1 \text{ lb}$ $F_B = 1 \text{ lb}$



Given $-A_x + F_B \cos(\theta) = 0$ $-A_y + F_B \sin(\theta) - W - L = 0$

$$F_B \cos(\theta)b + F_B \sin(\theta)c - Wa - L(d + c) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_B \end{pmatrix} = \text{Find}(A_x, A_y, F_B) \quad F_B = 4.19 \text{ kip} \quad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 3.208 \\ 1.967 \end{pmatrix} \text{ kip}$$

Örnek

If the wheelbarrow and its contents have a mass of M and center of mass at G , determine the magnitude of the resultant force which the man must exert on *each* of the two handles in order to hold the wheelbarrow in equilibrium.

Given:

$$M = 60 \text{ kg}$$

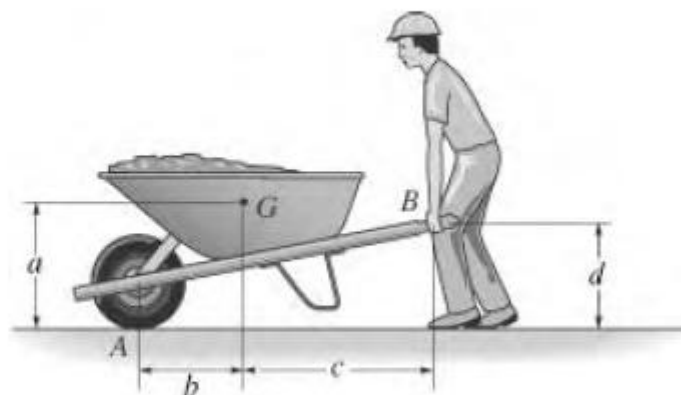
$$a = 0.6 \text{ m}$$

$$b = 0.5 \text{ m}$$

$$c = 0.9 \text{ m}$$

$$d = 0.5 \text{ m}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



Solution:

$$\curvearrowleft \Sigma M_B = 0; \quad -A_y (b + c) + M g c = 0$$

$$A_y = \frac{M g c}{b + c}$$

$$A_y = 378.386 \text{ N}$$

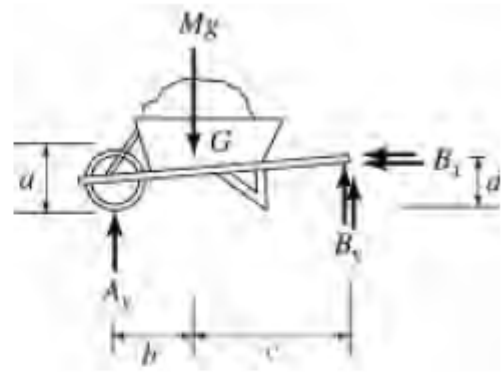
$$\rightarrow \Sigma F_x = 0; \quad B_x = 0 \text{ N}$$

$$B_x = 0$$

$$\uparrow \Sigma F_y = 0; \quad A_y - M g + 2 B_y = 0$$

$$B_y = \frac{M g - A_y}{2}$$

$$B_y = 105.107 \text{ N}$$



Örnek

The mobile crane has weight W_1 and center of gravity at G_1 ; the boom has weight W_2 and center of gravity at G_2 . Determine the smallest angle of tilt θ of the boom, without causing the crane to overturn if the suspended load has weight W . Neglect the thickness of the tracks at A and B .

Given:

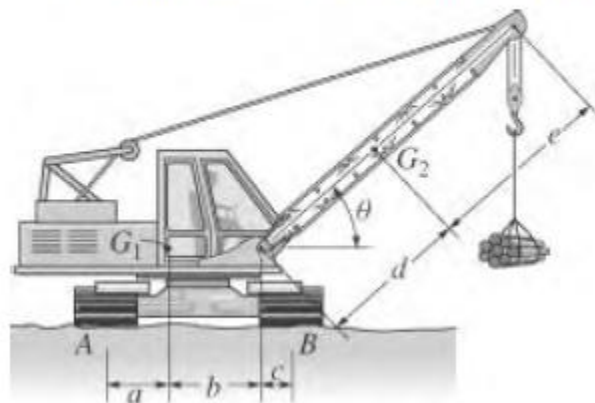
$$W_1 = 120000 \text{ lb}$$

$$W_2 = 30000 \text{ lb}$$

$$W = 40000 \text{ lb}$$

$$a = 4 \text{ ft}$$

$$b = 6 \text{ ft}$$



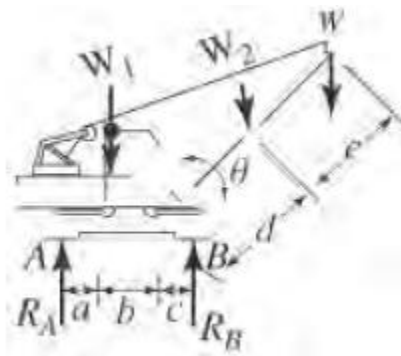
$$c = 3 \text{ ft}$$

$$d = 12 \text{ ft}$$

$$e = 15 \text{ ft}$$

Solution:

When tipping occurs, $R_A = 0$



$$\curvearrowleft \Sigma M_B = 0; \quad -W_2(d \cos(\theta) - c) - W[(d + e) \cos(\theta) - c] + W_1(b + c) = 0$$

$$\theta = \arccos \left[\frac{W_2 c + W c + W_1 (b + c)}{W_2 d + W (d + e)} \right]$$

$$\theta = 26.4 \text{ deg}$$

Örnek

The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB .

Given:

$$F_1 = 800 \text{ N}$$

$$F_2 = 350 \text{ N}$$

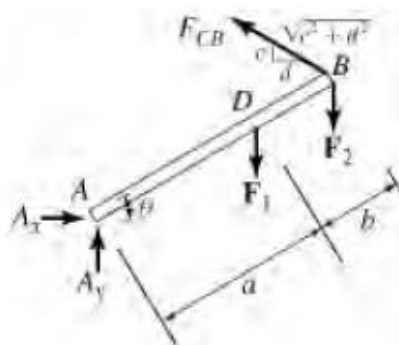
$$a = 1.5 \text{ m}$$

$$b = 1 \text{ m}$$

$$c = 3$$

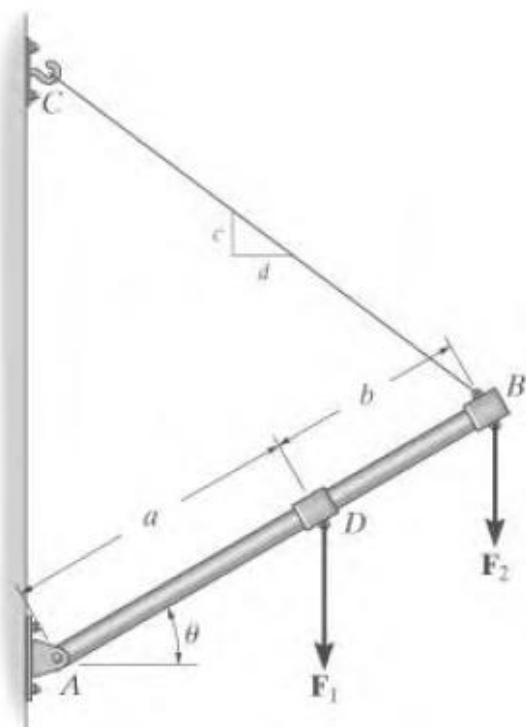
$$d = 4$$

$$\theta = 30 \text{ deg}$$



Solution:

$$\curvearrowleft \Sigma M_A = 0;$$



$$-F_1 a \cos(\theta) - F_2(a+b) \cos(\theta) + \frac{d}{\sqrt{c^2 + d^2}} F_{CB} (a+b) \sin(\theta) + \frac{c}{\sqrt{c^2 + d^2}} F_{CB}(a+b) \cos(\theta) = 0$$

$$F_{CB} = \frac{[F_1 a + F_2(a+b)] \cos(\theta) \sqrt{c^2 + d^2}}{d \sin(\theta)(a+b) + c \cos(\theta)(a+b)}$$

$$F_{CB} = 782 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - \frac{d}{\sqrt{c^2 + d^2}} F_{CB} = 0$$

$$A_x = \frac{d}{\sqrt{c^2 + d^2}} F_{CB}$$

$$A_x = 625 \text{ N}$$

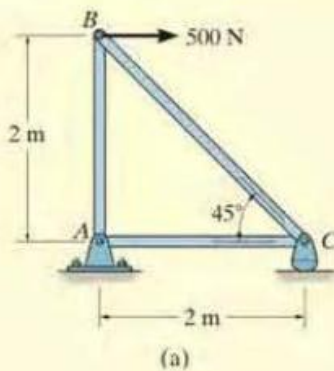
$$+\uparrow \Sigma F_y = 0; \quad A_y - F_1 - F_2 + \frac{c}{\sqrt{c^2 + d^2}} F_{CB} = 0$$

$$A_y = F_1 + F_2 - \frac{c}{\sqrt{c^2 + d^2}} F_{CB}$$

$$A_y = 681 \text{ N}$$

KAFES SİSTEMLERİ

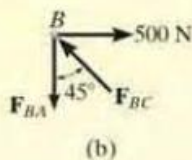
Örnek



Determine the force in each member of the truss shown in Fig. 6–8a and indicate whether the members are in tension or compression.

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint B.



Joint B. The free-body diagram of the joint at B is shown in Fig. 6–8b. Applying the equations of equilibrium, we have

$$\rightarrow \Sigma F_x = 0; \quad 500 \text{ N} - F_{BC} \sin 45^\circ = 0 \quad F_{BC} = 707.1 \text{ N (C) Ans.}$$

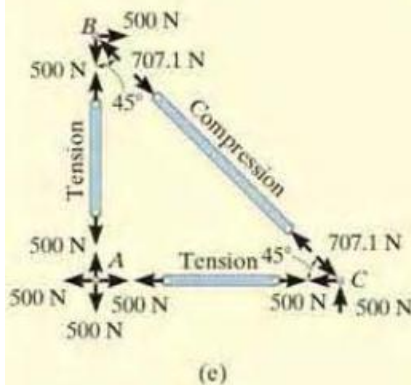
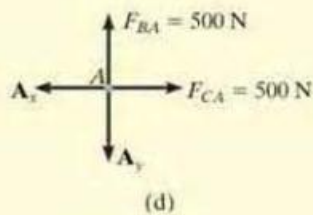
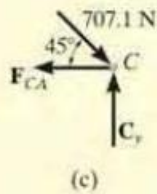


Fig. 6-8

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \cos 45^\circ - F_{BA} = 0 \quad F_{BA} = 500 \text{ N (T)} \quad \text{Ans.}$$

Since the force in member BC has been calculated, we can proceed to analyze joint C to determine the force in member CA and the support reaction at the rocker.

Joint C. From the free-body diagram of joint C , Fig. 6-8c, we have

$$\pm \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45^\circ \text{ N} = 0 \quad F_{CA} = 500 \text{ N (T)} \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45^\circ \text{ N} = 0 \quad C_y = 500 \text{ N} \quad \text{Ans.}$$

Joint A. Although it is not necessary, we can determine the components of the support reactions at joint A using the results of F_{CA} and F_{BA} . From the free-body diagram, Fig. 6-8d, we have

$$\pm \Sigma F_x = 0; \quad 500 \text{ N} - A_x = 0 \quad A_x = 500 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 500 \text{ N} - A_y = 0 \quad A_y = 500 \text{ N}$$

NOTE: The results of the analysis are summarized in Fig. 6-8e. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

Örnek

Determine the force in each member of the truss in Fig. 6-9a and indicate if the members are in tension or compression.

SOLUTION

Since joint C has one known and only two unknown forces acting on it, it is possible to start at this joint, then analyze joint D , and finally joint A . This way the support reactions will not have to be determined prior to starting the analysis.

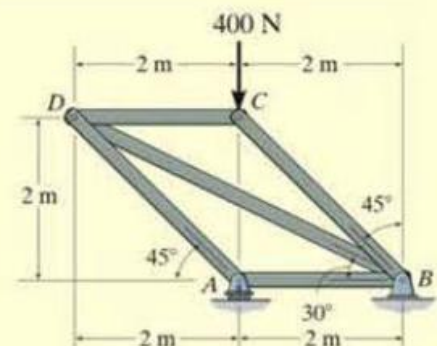
Joint C. By inspection of the force equilibrium, Fig. 6-9b, it can be seen that both members BC and CD must be in compression.

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 45^\circ - 400 \text{ N} = 0$$

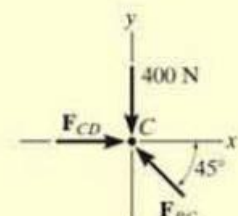
$$F_{BC} = 565.69 \text{ N} = 566 \text{ N (C)} \quad \text{Ans.}$$

$$\pm \Sigma F_x = 0; \quad F_{CD} - (565.69 \text{ N}) \cos 45^\circ = 0$$

$$F_{CD} = 400 \text{ N (C)} \quad \text{Ans.}$$



(a)



Joint D. Using the result $F_{CD} = 400 \text{ N (C)}$, the force in members BD and AD can be found by analyzing the equilibrium of joint D . We will assume F_{AD} and F_{BD} are both tensile forces, Fig. 6-9c. The x' , y' coordinate system will be established so that the x' axis is directed along F_{BD} . This way, we will eliminate the need to solve two equations simultaneously. Now F_{AD} can be obtained *directly* by applying $\Sigma F_{y'} = 0$.

$$+\nearrow \Sigma F_{y'} = 0; \quad -F_{AD} \sin 15^\circ - 400 \sin 30^\circ = 0$$

$$F_{AD} = -772.74 \text{ N} = 773 \text{ N (C)} \quad \text{Ans.}$$

The negative sign indicates that F_{AD} is a compressive force. Using this result,

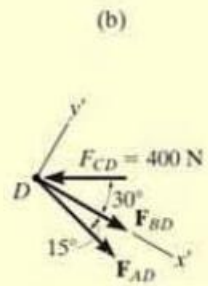
$$+\searrow \Sigma F_{x'} = 0; \quad F_{BD} + (-772.74 \cos 15^\circ) - 400 \cos 30^\circ = 0$$

$$F_{BD} = 1092.82 \text{ N} = 1.09 \text{ kN (T)} \quad \text{Ans.}$$

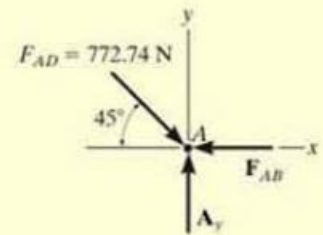
Joint A. The force in member AB can be found by analyzing the equilibrium of joint A , Fig. 6-9d. We have

$$\pm \Sigma F_x = 0; \quad (772.74 \text{ N}) \cos 45^\circ - F_{AB} = 0$$

$$F_{AB} = 546.41 \text{ N (C)} = 546 \text{ N (C)} \quad \text{Ans.}$$



(c)



(d)

Fig. 6-9

Örnek

Determine the force in each member of the truss shown in Fig. 6-10a. Indicate whether the members are in tension or compression.

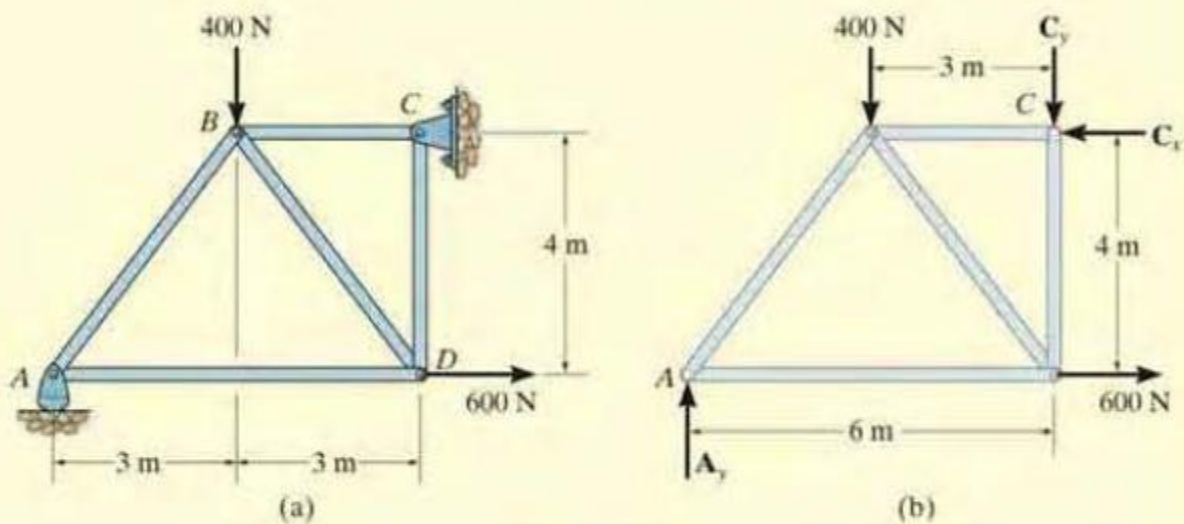
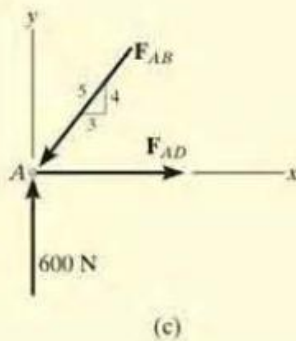


Fig. 6-10

SOLUTION

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has more than three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. 6-10b. Applying the equations of equilibrium, we have

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad 600 \text{ N} - C_x &= 0 & C_x &= 600 \text{ N} \\ \curvearrowright + \Sigma M_C = 0; \quad -A_y(6 \text{ m}) + 400 \text{ N}(3 \text{ m}) + 600 \text{ N}(4 \text{ m}) &= 0 \\ & A_y = 600 \text{ N} \\ + \uparrow \Sigma F_y = 0; \quad 600 \text{ N} - 400 \text{ N} - C_y &= 0 & C_y &= 200 \text{ N} \end{aligned}$$



The analysis can now start at either joint *A* or *C*. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.

Joint A. (Fig. 6-10c). As shown on the free-body diagram, F_{AB} is assumed to be compressive and F_{AD} is tensile. Applying the equations of equilibrium, we have

$$\begin{aligned} + \uparrow \Sigma F_y = 0; \quad 600 \text{ N} - \frac{4}{5}F_{AB} &= 0 & F_{AB} &= 750 \text{ N (C)} & \text{Ans.} \\ \rightarrow \Sigma F_x = 0; \quad F_{AD} - \frac{3}{5}(750 \text{ N}) &= 0 & F_{AD} &= 450 \text{ N (T)} & \text{Ans.} \end{aligned}$$

Joint D. (Fig. 6-10d). Using the result for F_{AD} and summing forces in the horizontal direction, Fig. 6-10d, we have

$$\rightarrow \Sigma F_x = 0; \quad -450 \text{ N} + \frac{3}{5}F_{DB} + 600 \text{ N} = 0 \quad F_{DB} = -250 \text{ N}$$

The negative sign indicates that F_{DB} acts in the *opposite sense* to that shown in Fig. 6-10d.* Hence,

$$F_{DB} = 250 \text{ N (T)} \quad \text{Ans.}$$

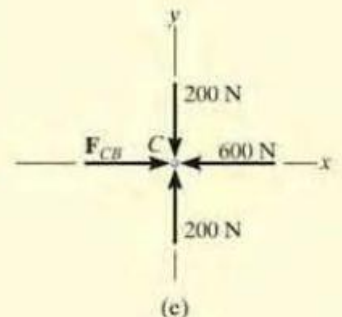
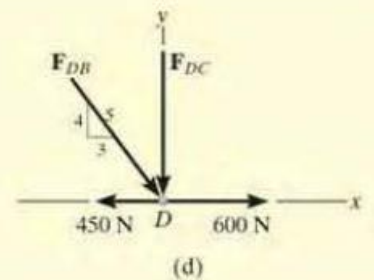
To determine F_{DC} , we can either correct the sense of F_{DB} on the free-body diagram, and then apply $\Sigma F_y = 0$, or apply this equation and retain the negative sign for F_{DB} , i.e.,

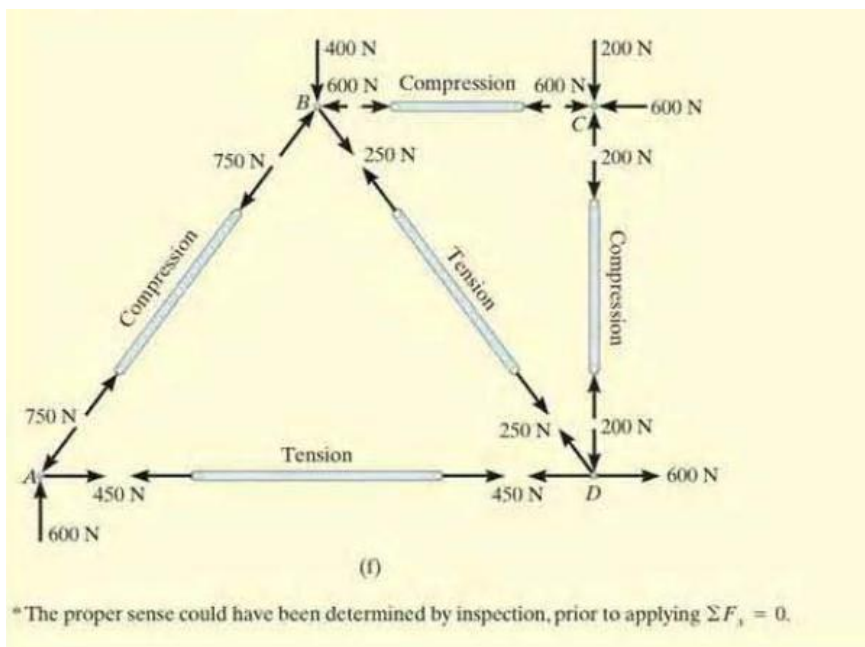
$$+ \uparrow \Sigma F_y = 0; \quad -F_{DC} - \frac{4}{5}(-250 \text{ N}) = 0 \quad F_{DC} = 200 \text{ N (C)} \quad \text{Ans.}$$

Joint C. (Fig. 6-10e).

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad F_{CB} - 600 \text{ N} &= 0 & F_{CB} &= 600 \text{ N (C)} & \text{Ans.} \\ + \uparrow \Sigma F_y = 0; \quad 200 \text{ N} - 200 \text{ N} &= 0 & & \text{(check)} \end{aligned}$$

NOTE: The analysis is summarized in Fig. 6-10f, which shows the free-body diagram for each joint and member.





Örnek

The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$P_1 = 800 \text{ lb}$$

$$P_2 = 0 \text{ lb}$$

$$a = 4 \text{ ft}$$

$$\theta = 45^\circ$$

Solution:

Initial Guesses

$$F_{AB} = 1 \text{ lb} \quad F_{AD} = 1 \text{ lb} \quad F_{DC} = 1 \text{ lb}$$

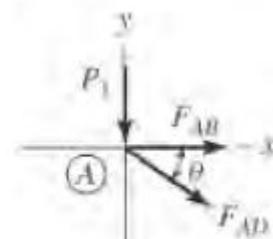
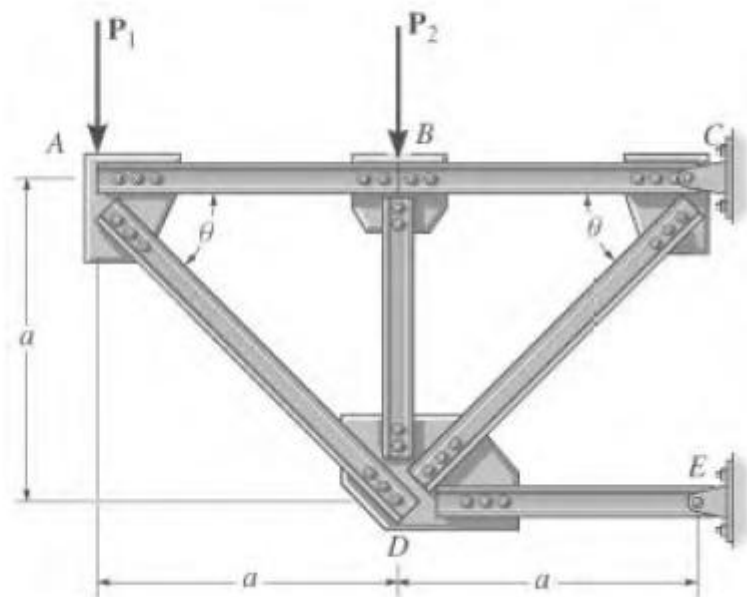
$$F_{BC} = 1 \text{ lb} \quad F_{BD} = 1 \text{ lb} \quad F_{DE} = 1 \text{ lb}$$

Given

$$\text{Joint A:} \quad F_{AB} + F_{AD} \cos(\theta) = 0$$

$$-P_1 - F_{AD} \sin(\theta) = 0$$

$$\text{Joint B:} \quad F_{BC} - F_{AB} = 0$$



Joint B: $F_{BC} - F_{AB} = 0$

$-P_2 - F_{BD} = 0$

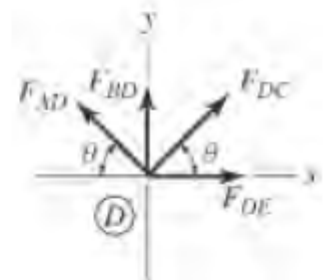
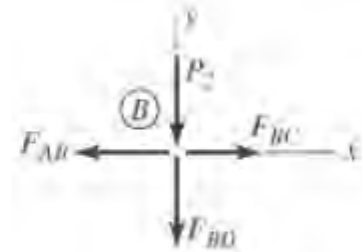
Joint D: $(F_{DC} - F_{AD})\cos(\theta) + F_{DE} = 0$

$(F_{DC} + F_{AD})\sin(\theta) + F_{BD} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{BC}, F_{BD}, F_{DC}, F_{DE})$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 800 \\ -1131 \\ 800 \\ 0 \\ 1131 \\ -1600 \end{pmatrix} \text{ lb}$$

Positive means Tension,
Negative means Compression



Örnek

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3 \text{ N}$

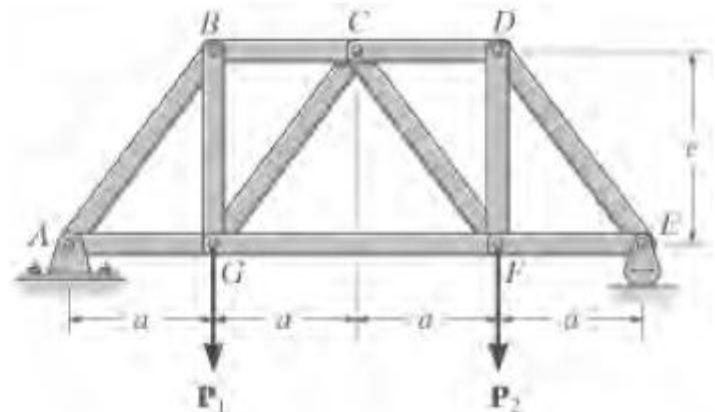
Given:

$P_1 = 20 \text{ kN}$

$P_2 = 10 \text{ kN}$

$a = 1.5 \text{ m}$

$e = 2 \text{ m}$



Solution: $\theta = \text{atan}\left(\frac{e}{a}\right)$

Initial Guesses:

$$F_{AB} = 1 \text{ kN} \quad F_{AG} = 1 \text{ kN} \quad F_{CF} = 1 \text{ kN}$$

$$F_{BC} = 1 \text{ kN} \quad F_{BG} = 1 \text{ kN} \quad F_{DE} = 1 \text{ kN}$$

$$F_{CG} = 1 \text{ kN} \quad F_{FG} = 1 \text{ kN} \quad F_{EF} = 1 \text{ kN}$$

$$F_{CD} = 1 \text{ kN} \quad F_{DF} = 1 \text{ kN}$$

Given

Joint B $F_{BC} - F_{AB} \cos(\theta) = 0$

$$-F_{BG} - F_{AB} \sin(\theta) = 0$$

Joint G $F_{FG} + F_{CG} \cos(\theta) - F_{AG} = 0$

$$F_{CG} \sin(\theta) + F_{BG} - P_1 = 0$$

Joint C $-F_{BC} + F_{CD} + (F_{CF} - F_{CG}) \cos(\theta) = 0$

$$-(F_{CG} + F_{CF}) \sin(\theta) = 0$$

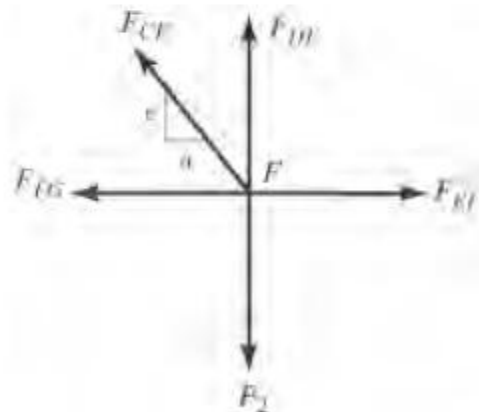
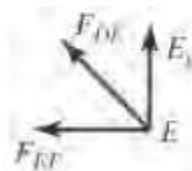
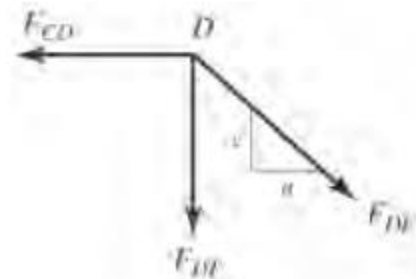
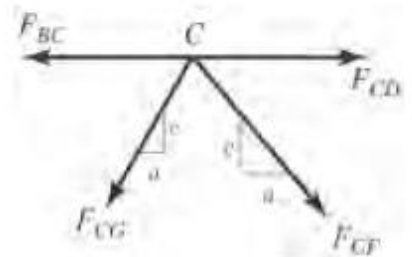
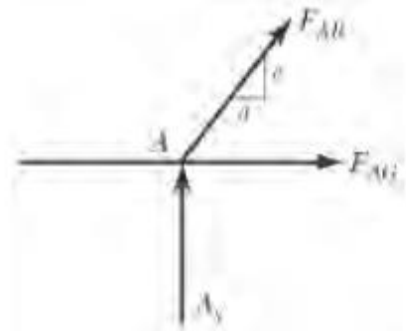
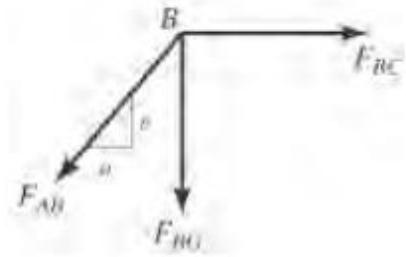
Joint D $-F_{CD} + F_{DE} \cos(\theta) = 0$

$$-F_{DF} - F_{DE} \sin(\theta) = 0$$

Joint F $F_{EF} - F_{FG} - F_{CF} \cos(\theta) = 0$

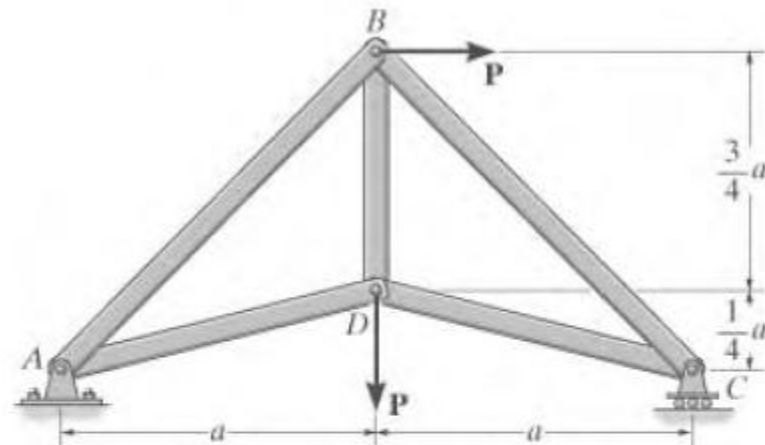
$$F_{DF} + F_{CF} \sin(\theta) - P_2 = 0$$

Joint E $-F_{DE} \cos(\theta) - F_{EF} = 0$



Örnek

Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression.



Solution:

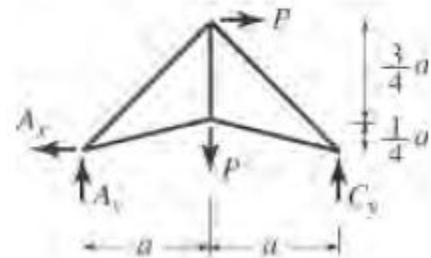
$$\Sigma M_A = 0; \quad -P a + C_y 2a - P a = 0$$

$$C_y = P$$

Joint C:

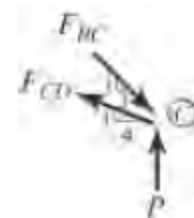
$$\Sigma F_x = 0; \quad \frac{1}{\sqrt{2}} F_{BC} - \frac{4}{\sqrt{17}} F_{CD} = 0$$

$$\Sigma F_y = 0; \quad P + \frac{1}{\sqrt{17}} F_{CD} - \frac{1}{\sqrt{2}} F_{BC} = 0$$



$$F_{BC} = \frac{4\sqrt{2}P}{3} = 1.886 P \quad (\text{C})$$

$$F_{CD} = \frac{\sqrt{17}P}{3} = 1.374 P \quad (\text{T})$$



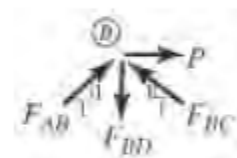
Joint B:

$$\Sigma F_x = 0; \quad P - \frac{1}{\sqrt{2}} F_{CD} + \frac{1}{\sqrt{2}} F_{AB} = 0$$

$$\Sigma F_y = 0; \quad \frac{1}{\sqrt{2}} F_{CD} + \frac{1}{\sqrt{2}} F_{AB} - F_{BD} = 0$$

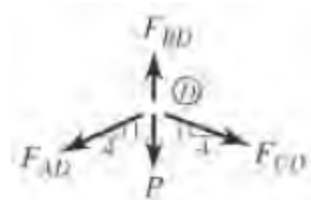
$$F_{AB} = \frac{\sqrt{2}P}{3} = 0.471 P \quad (\text{C})$$

$$F_{BD} = \frac{5P}{3} = 1.667 P \quad (\text{T})$$



Joint D:

$$\Sigma F_x = 0; \quad F_{DA} = F_{CD} = 1.374 P \quad (\text{T})$$



Örnek

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The horizontal force component at *A* must be zero. Why?

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$F_1 = 600 \text{ lb}$$

$$F_2 = 800 \text{ lb}$$

$$a = 4 \text{ ft}$$

$$b = 3 \text{ ft}$$

$$\theta = 60^\circ$$

Solution:

Initial Guesses

$$F_{BA} = 1 \text{ lb} \quad F_{BD} = 1 \text{ lb} \quad F_{CB} = 1 \text{ lb} \quad F_{CD} = 1 \text{ lb}$$

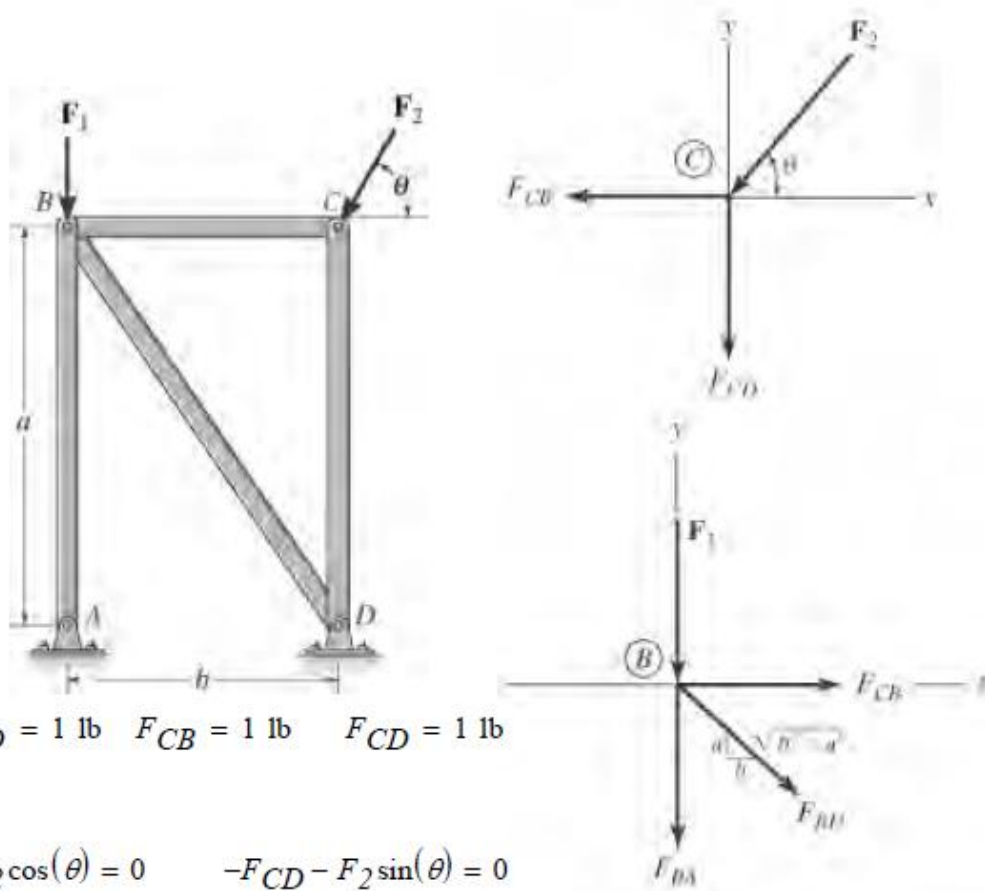
Given

$$\text{Joint C} \quad -F_{CB} - F_2 \cos(\theta) = 0 \quad -F_{CD} - F_2 \sin(\theta) = 0$$

$$\text{Joint B} \quad F_{CB} + F_{BD} \frac{b}{\sqrt{a^2 + b^2}} = 0 \quad -F_{BA} - F_{BD} \frac{a}{\sqrt{a^2 + b^2}} - F_1 = 0$$

$$\begin{pmatrix} F_{BA} \\ F_{BD} \\ F_{CB} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{BA}, F_{BD}, F_{CB}, F_{CD}) \quad \begin{pmatrix} F_{BA} \\ F_{BD} \\ F_{CB} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -1.133 \times 10^3 \\ 666.667 \\ -400 \\ -692.82 \end{pmatrix}$$

Positive means Tension
Negative means Compression



Örnek

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The resultant force at the pin *E* acts along member *ED*. Why?

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$F_1 = 3 \text{ kN}$$

$$F_2 = 2 \text{ kN}$$

$$a = 3 \text{ m}$$

$$b = 4 \text{ m}$$

Solution:

Initial Guesses:

$$F_{CB} = 1 \text{ kN} \quad F_{CD} = 1 \text{ kN} \quad F_{BA} = 1 \text{ kN}$$

$$F_{BD} = 1 \text{ kN} \quad F_{DA} = 1 \text{ kN} \quad F_{DE} = 1 \text{ kN}$$

Given

$$\text{Joint C} \quad -F_{CB} - F_{CD} \frac{2a}{\sqrt{(2a)^2 + b^2}} = 0$$

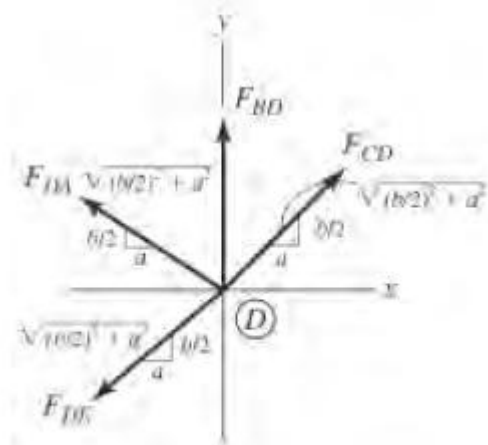
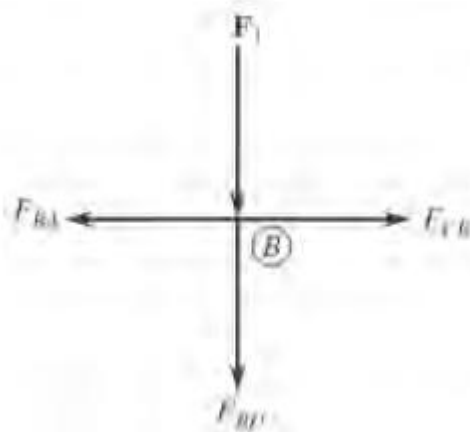
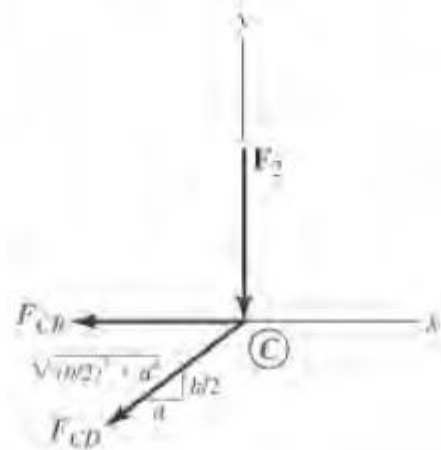
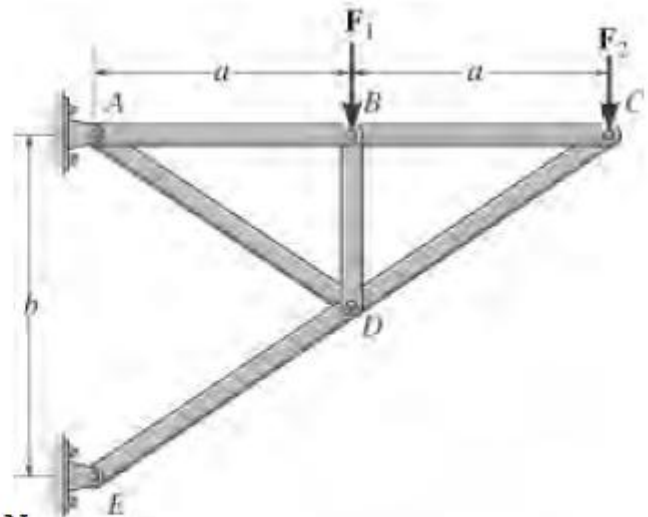
$$-F_2 - F_{CD} \frac{b}{\sqrt{(2a)^2 + b^2}} = 0$$

$$\text{Joint B} \quad -F_{BA} + F_{CB} = 0$$

$$-F_1 - F_{BD} = 0$$

$$\text{Joint D} \quad (F_{CD} - F_{DA} - F_{DE}) \frac{2a}{\sqrt{(2a)^2 + b^2}} = 0$$

$$F_{BD} + (F_{CD} + F_{DA} - F_{DE}) \frac{b}{\sqrt{(2a)^2 + b^2}} = 0$$



$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{CB}, F_{CD}, F_{BA}, F_{BD}, F_{DA}, F_{DE})$$

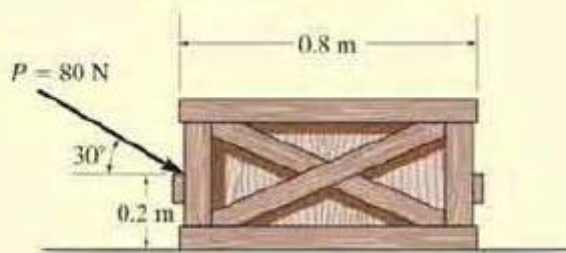
$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 3 \\ -3.606 \\ 3 \\ -3 \\ 2.704 \\ -6.31 \end{pmatrix} \text{ kN}$$

Positive means Tension,
Negative means Compression

SÜRTÜNME

Örnek

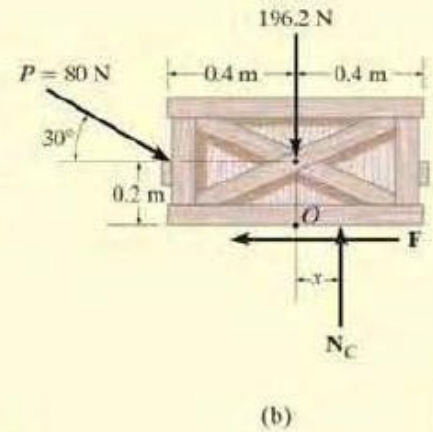
The uniform crate shown in Fig. 8–7a has a mass of 20 kg. If a force $P = 80 \text{ N}$ is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.



(a)

SOLUTION

Free-Body Diagram. As shown in Fig. 8-7b, the *resultant* normal force N_C must act a distance x from the crate's center line in order to counteract the tipping effect caused by P . There are *three unknowns*, F , N_C , and x , which can be determined strictly from the *three* equations of equilibrium.



Equations of Equilibrium.

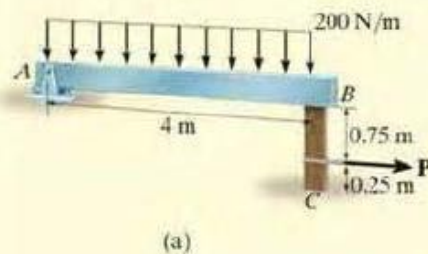
$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & 80 \cos 30^\circ \text{ N} - F &= 0 \\ +\uparrow \Sigma F_y &= 0; & -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} &= 0 \\ \zeta + \Sigma M_O &= 0; & 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) &= 0 \end{aligned}$$

Solving,

$$\begin{aligned} F &= 69.3 \text{ N} \\ N_C &= 236 \text{ N} \\ x &= -0.00908 \text{ m} = -9.08 \text{ mm} \end{aligned}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4 \text{ m}$. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{\max} = \mu_s N_C = 0.3(236 \text{ N}) = 70.8 \text{ N}$. Since $F = 69.3 \text{ N} < 70.8 \text{ N}$, the crate will *not slip*, although it is very close to doing so.

Örnek



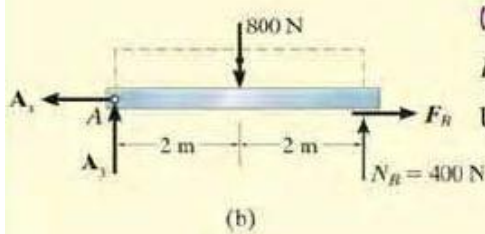
Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC , Fig. 8-10a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force P needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.

SOLUTION

Free-Body Diagrams. The free-body diagram of the beam is shown in Fig. 8-10b. Applying $\Sigma M_A = 0$, we obtain $N_B = 400 \text{ N}$. This result is shown on the free-body diagram of the post, Fig. 8-10c. Referring to this member, the *four unknowns* F_B , P , F_C , and N_C are determined from the *three* equations of equilibrium and *one* frictional equation applied either at B or C .

Equations of Equilibrium and Friction.

$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & P - F_B - F_C &= 0 & (1) \\ +\uparrow \Sigma F_y &= 0; & N_C - 400 \text{ N} &= 0 & (2) \\ \zeta + \Sigma M_C &= 0; & -P(0.25 \text{ m}) + F_B(1 \text{ m}) &= 0 & (3) \end{aligned}$$



(Post Slips at B and Rotates about C.) This requires $F_C \leq \mu_C N_C$ and

$$F_B = \mu_B N_B; \quad F_B = 0.2(400 \text{ N}) = 80 \text{ N}$$

Using this result and solving Eqs. 1 through 3, we obtain

$$P = 320 \text{ N}$$

$$F_C = 240 \text{ N}$$

$$N_C = 400 \text{ N}$$

Since $F_C = 240 \text{ N} > \mu_C N_C = 0.5(400 \text{ N}) = 200 \text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at C and Rotates about B.) Here $F_B \leq \mu_B N_B$ and

$$F_C = \mu_C N_C; \quad F_C = 0.5 N_C \quad (4)$$

Solving Eqs. 1 through 4 yields

$$P = 267 \text{ N}$$

Ans.

$$N_C = 400 \text{ N}$$

$$F_C = 200 \text{ N}$$

$$F_B = 66.7 \text{ N}$$

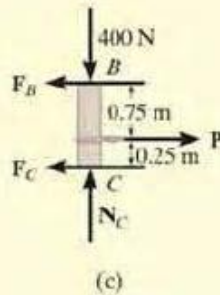


Fig. 8-10

Obviously, this case occurs first since it requires a *smaller* value for P .

Örnek

The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is μ_s and a torque M is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) P_1 (b) P_2 .

Given:

$$\mu_s = 0.3$$

$$M = 5 \text{ N} \cdot \text{m}$$

$$a = 50 \text{ mm}$$

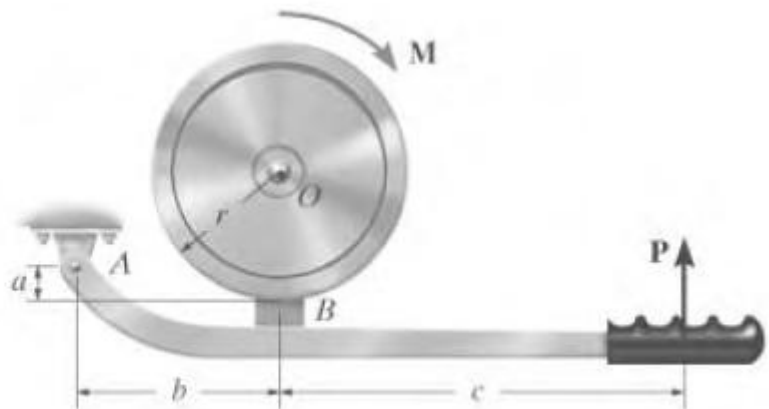
$$b = 200 \text{ mm}$$

$$c = 400 \text{ mm}$$

$$r = 150 \text{ mm}$$

$$P_1 = 30 \text{ N}$$

$$P_2 = 70 \text{ N}$$



Solution: To hold lever:

$$\sum M_O = 0; \quad F_B r - M = 0$$

$$F_B = \frac{M}{r} \quad F_B = 33.333 \text{ N}$$

Require $N_B = \frac{F_B}{\mu_s} \quad N_B = 111.1 \text{ N}$

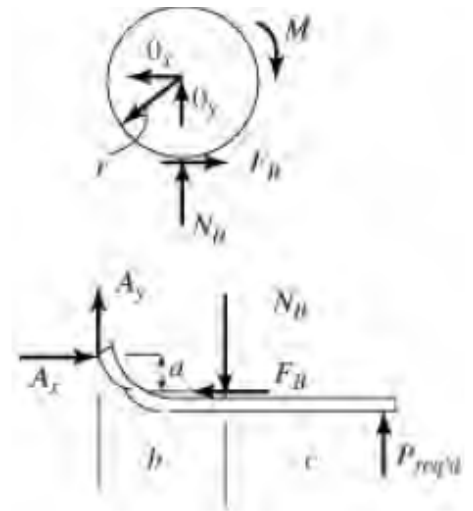
Lever,

$$\sum M_A = 0; \quad P_{Reqd} (b + c) - N_B b - F_B a = 0$$

$$P_{Reqd} = \frac{N_B b + F_B a}{b + c} \quad P_{Reqd} = 39.8 \text{ N}$$

(a) If $P_1 = 30.00 \text{ N} > P_{Reqd} = 39.81 \text{ N}$ then the break will hold the wheel

(b) If $P_2 = 70.00 \text{ N} > P_{Reqd} = 39.81 \text{ N}$ then the break will hold the wheel



Örnek

The fork lift has a weight W_1 and center of gravity at G . If the rear wheels are powered, whereas the front wheels are free to roll, determine the maximum number of crates, each of weight W_2 that the fork lift can push forward. The coefficient of static friction between the wheels and the ground is μ_s and between each crate and the ground is μ'_s .

Given:

$$W_1 = 2400 \text{ lb}$$

$$W_2 = 300 \text{ lb}$$

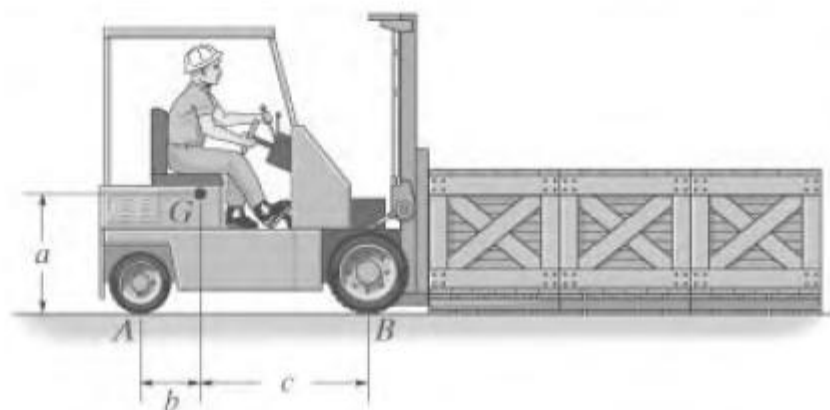
$$\mu_s = 0.4$$

$$\mu'_s = 0.35$$

$$a = 2.5 \text{ ft}$$

$$b = 1.25 \text{ ft}$$

$$c = 3.50 \text{ ft}$$



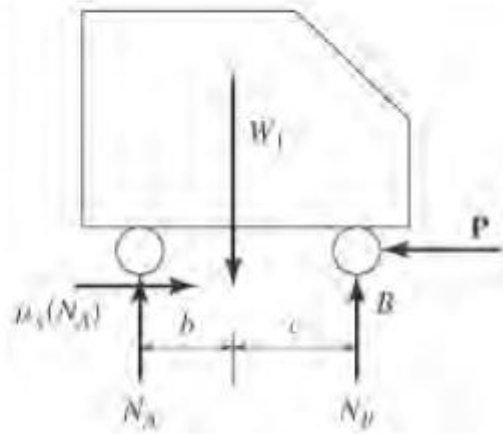
Solution:

Fork lift:

$$\Sigma M_B = 0; \quad W_I c - N_A (b + c) = 0$$

$$N_A = W_I \left(\frac{c}{b + c} \right) \quad N_A = 1768.4 \text{ lb}$$

$$\Sigma F_x = 0; \quad \mu_s N_A - P = 0$$



$$P = \mu_s N_A \quad P = 707.37 \text{ lb}$$

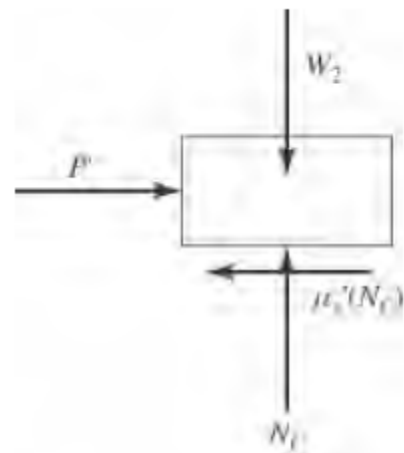
Crate:

$$\Sigma F_y = 0; \quad N_C - W_2 = 0$$

$$N_C = W_2 \quad N_C = 300.00 \text{ lb}$$

$$\Sigma F_x = 0; \quad P' - \mu'_s N_C = 0$$

$$P' = \mu'_s N_C \quad P' = 105.00 \text{ lb}$$



$$\text{Thus} \quad n = \frac{P}{P'} \quad n = 6.74 \quad n = \text{floor}(n) \quad n = 6.00$$

Örnek

The crate has a weight W and a center of gravity at G . Determine the horizontal force P required to tow it. Also, determine the location of the resultant normal force measured from A .

Given:

$$a = 3.5 \text{ ft}$$

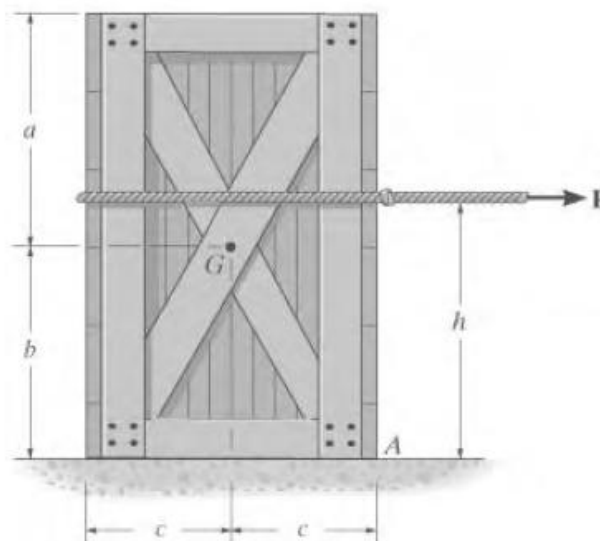
$$b = 3 \text{ ft}$$

$$c = 2 \text{ ft}$$

$$W = 200 \text{ lb}$$

$$h = 4 \text{ ft}$$

$$\mu_s = 0.4$$



$$\Sigma F_x = 0; \quad P = F_O$$

$$\Sigma F_y = 0; \quad N_O = W$$

$$N_O = 200.00 \text{ lb}$$

$$\Sigma M_O = 0; \quad -P h + W x = 0$$

$$F_O = \mu_s N_O$$

$$F_O = 80.00 \text{ lb}$$

$$P = F_O$$

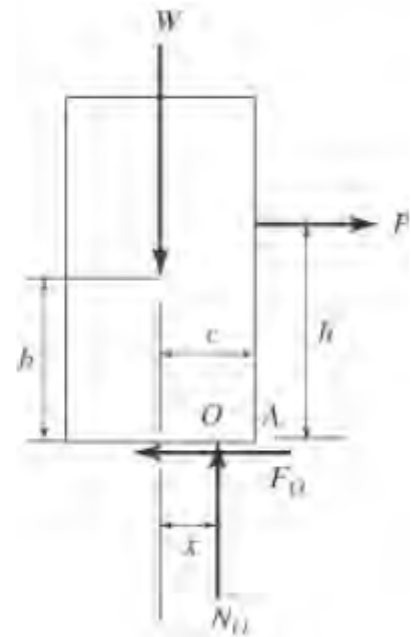
$$P = 80.00 \text{ lb}$$

$$x = P \frac{h}{W}$$

$$x = 1.60 \text{ ft}$$

The distance of N_O from A is

$$c - x = 0.40 \text{ ft}$$



AĞIRLIK MERKEZİ

Örnek

A rack is made from roll-formed sheet steel and has the cross section shown. Determine the location (x_c, y_c) of the centroid of the cross section. The dimensions are indicated at the center thickness of each segment.

Given:

$$a = 15 \text{ mm}$$

$$c = 80 \text{ mm}$$

$$d = 50 \text{ mm}$$

$$e = 30 \text{ mm}$$

Solution:

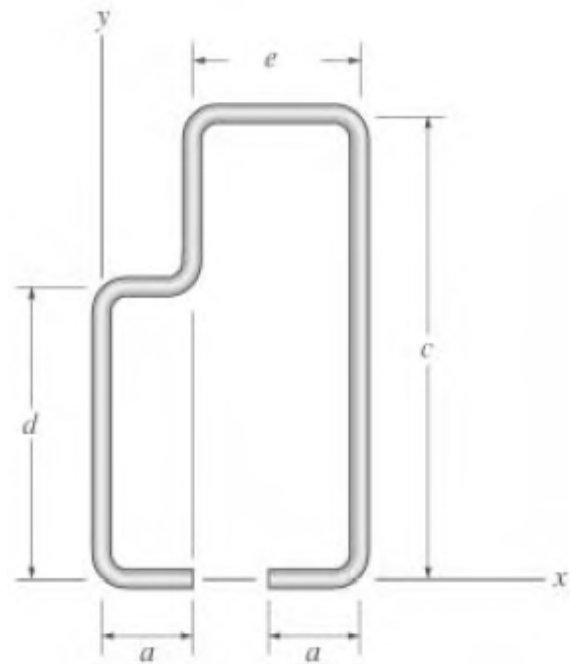
$$L = 3a + 2c + e$$

$$x_C = \frac{2a \frac{a}{2} + a \left(e + \frac{a}{2} \right) + c(a + e) + e \left(a + \frac{e}{2} \right) + (c - d)a}{L}$$

$$x_C = 24.4 \text{ mm}$$

$$y_C = \frac{d \frac{d}{2} + c \frac{c}{2} + (c - d) \frac{d + c}{2} + ad + ec}{L}$$

$$y_C = 40.6 \text{ mm}$$



Örnek

The three members of the frame each have weight density γ . Locate the position (x_C, y_C) of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the fixed support A .

Given:

$$\gamma = 4 \frac{\text{lb}}{\text{ft}}$$

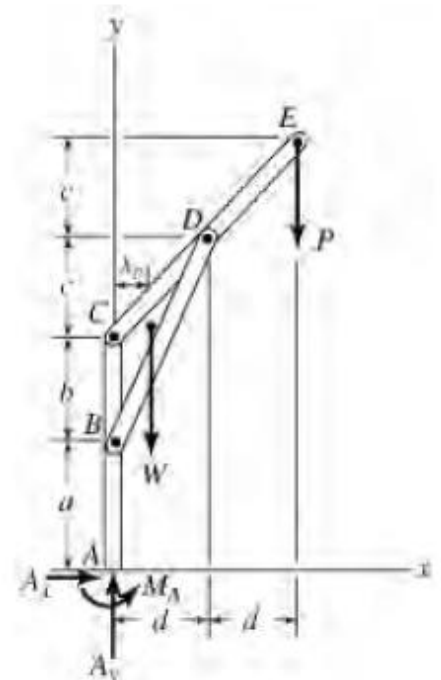
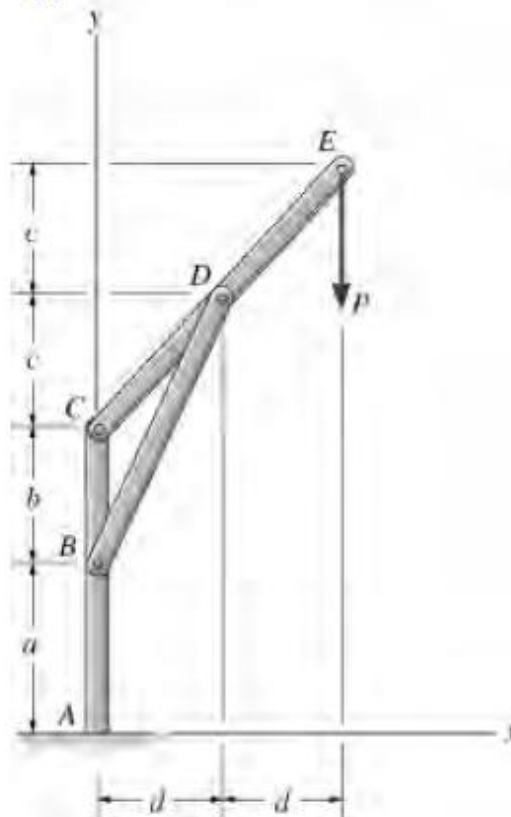
$$P = 60 \text{ lb}$$

$$a = 4 \text{ ft}$$

$$b = 3 \text{ ft}$$

$$c = 3 \text{ ft}$$

$$d = 3 \text{ ft}$$



Solution:

$$W = \gamma \sqrt{d^2 + (b+c)^2} + \gamma 2 \sqrt{d^2 + c^2} + \gamma(a+b) \quad W = 88.774 \text{ lb}$$

$$x_c = \frac{\gamma \sqrt{d^2 + (b+c)^2} \frac{d}{2} + \gamma 2 \sqrt{d^2 + c^2} d}{W} \quad x_c = 1.6 \text{ ft}$$

$$y_c = \frac{\gamma(a+b) \left(\frac{a+b}{2} \right) + \gamma \sqrt{d^2 + (b+c)^2} \left(a + \frac{b+c}{2} \right) + \gamma 2 \sqrt{d^2 + c^2} (a+b+c)}{W}$$

$$y_c = 7.043 \text{ ft}$$

Equilibrium

$$A_x = 0 \quad A_x = 0 \text{ lb} \quad A_x = 0 \text{ lb}$$

$$A_y - W - P = 0 \quad A_y = W + P \quad A_y = 148.8 \text{ lb}$$

$$M_A - W x_c - P 2d = 0 \quad M_A = W x_c + P 2d \quad M_A = 502 \text{ lb ft}$$

Örnek

Locate the centroid (x_c, y_c) of the shaded area.

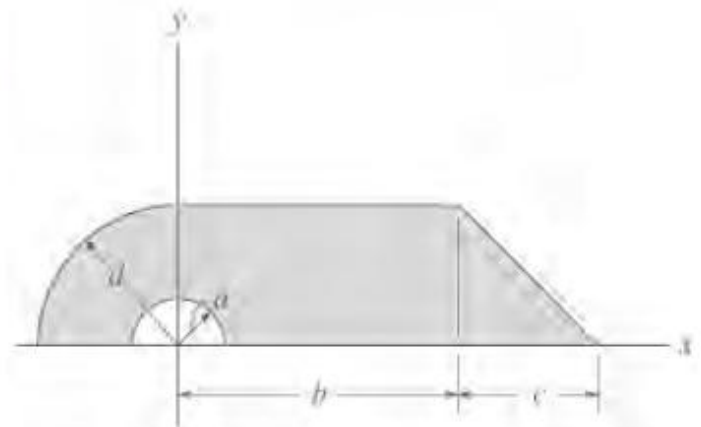
Given:

$$a = 1 \text{ in}$$

$$b = 6 \text{ in}$$

$$c = 3 \text{ in}$$

$$d = 3 \text{ in}$$



Solution:

$$A = b d + \frac{\pi d^2}{4} - \frac{\pi a^2}{2} + \frac{1}{2}(d c)$$

$$x_c = \frac{1}{A} \left[b d \frac{b}{2} - \frac{\pi d^2}{4} \left(\frac{4d}{3\pi} \right) + \frac{1}{2} d c \left(b + \frac{c}{3} \right) \right]$$

$$x_c = 2.732 \text{ in}$$

$$y_c = \frac{1}{A} \left[b d \left(\frac{d}{2} \right) + \frac{\pi d^2}{4} \left(\frac{4d}{3\pi} \right) - \frac{\pi a^2}{2} \left(\frac{4a}{3\pi} \right) + \frac{1}{2} d c \left(\frac{d}{3} \right) \right]$$

$$y_c = 1.423 \text{ in}$$

Örnek

The tank and compressor have a mass M_T and mass center at G_T and the motor has a mass M_M and a mass center at G_M . Determine the angle of tilt, θ , of the tank so that the unit will be on the verge of tipping over.

Given:

$$a = 300 \text{ mm}$$

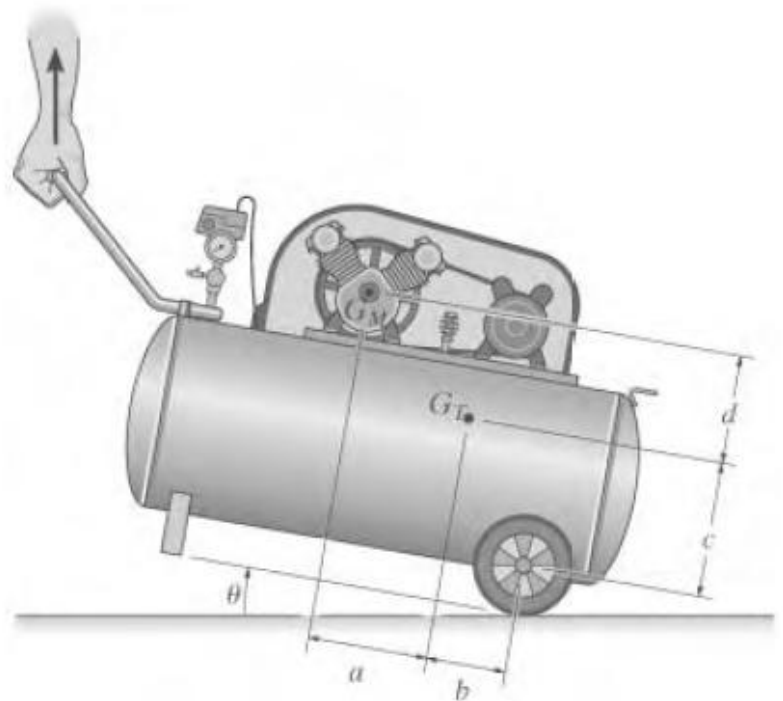
$$b = 200 \text{ mm}$$

$$c = 350 \text{ mm}$$

$$d = 275 \text{ mm}$$

$$M_T = 15 \text{ kg}$$

$$M_M = 70 \text{ kg}$$



Solution:

$$x_c = \frac{b M_T + (a + b) M_M}{M_T + M_M}$$

$$x_c = 0.4471 \text{ m}$$

$$y_c = \frac{c M_T + (c + d) M_M}{M_T + M_M}$$

$$y_c = 0.57647 \text{ m}$$

$$\theta = \text{atan} \left(\frac{x_c}{y_c} \right)$$

$$\theta = 37.8 \text{ deg}$$

