Kinetics of a Particle: Force and Acceleration

13

CHAPTER OBJECTIVES

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

13.1 Newton's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force **F** to a particle, and then measuring the acceleration **a**. Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio $m = F/a$. This positive scalar m is called the *mass* of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

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If the mass of the particle is m, Newton's second law of motion may be written in mathematical form as

$$\mathbf{F} = m\mathbf{a}$$

The above equation, which is referred to as the *equation of motion*, is one of the most important formulations in mechanics.* As previously stated, its validity is based solely on *experimental evidence*. In 1905, however, Albert Einstein developed the theory of relativity and placed limitations on the use of Newton's second law for describing general particle motion. Through experiments it was proven that *time* is not an absolute quantity as assumed by Newton; and as a result, the equation of motion fails to predict the exact behavior of a particle, especially when the particle's speed approaches the speed of light (0.3 Gm/s). Developments of the theory of quantum mechanics by Erwin Schrödinger and others indicate further that conclusions drawn from using this equation are also invalid when particles are the size of an atom and move close to one another. For the most part, however, these requirements regarding particle speed and size are not encountered in engineering problems, so their effects will not be considered in this book.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as

$$F = G\frac{m_1 m_2}{r^2} \tag{13–1}$$

where

F = force of attraction between the two particles

G = universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

m_1, m_2 = mass of each of the two particles

r = distance between the centers of the two particles

*Since m is constant, we can also write $\mathbf{F} = d(m\mathbf{v})/dt$, where $m\mathbf{v}$ is the particle's linear momentum. Here the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

In the case of a particle located at or near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the particle. This force is termed the "weight" and, for our purpose, it will be the only gravitational force considered.

From Eq. 13–1, we can develop a general expression for finding the weight W of a particle having a mass $m_1 = m$. Let $m_2 = M_e$ be the mass of the earth and r the distance between the earth's center and the particle. Then, if $g = GM_e/r^2$, we have

$$W = mg$$

By comparison with $F = ma$, we term g the acceleration due to gravity. For most engineering calculations g is a point on the surface of the earth at sea level, and at a latitude of 45°, which is considered the "standard location." Here the values $g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$ will be used for calculations.

In the SI system the mass of the body is specified in kilograms, and the weight must be calculated using the above equation, Fig. 13–1*a*. Thus,

$$W = mg \text{ (N)} \qquad (g = 9.81 \text{ m/s}^2) \tag{13–2}$$

As a result, a body of mass 1 kg has a weight of 9.81 N; a 2-kg body weighs 19.62 N; and so on.

In the FPS system the weight of the body is specified in pounds. The mass is measured in slugs, a term derived from "sluggish" which refers to the body's inertia. It must be calculated, Fig. 13–1*b*, using

$$m = \frac{W}{g} \text{ (slug)} \qquad (g = 32.2 \text{ ft/s}^2) \tag{13–3}$$

Therefore, a body weighing 32.2 lb has a mass of 1 slug; a 64.4-lb body has a mass of 2 slugs; and so on.

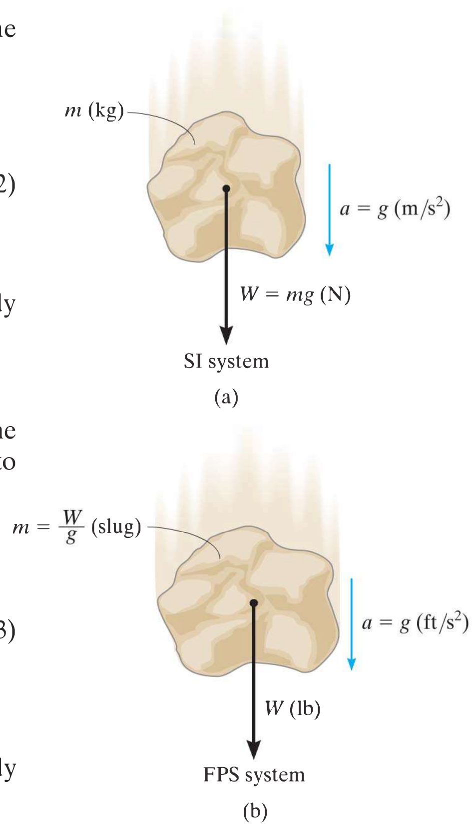


Fig. 13–1

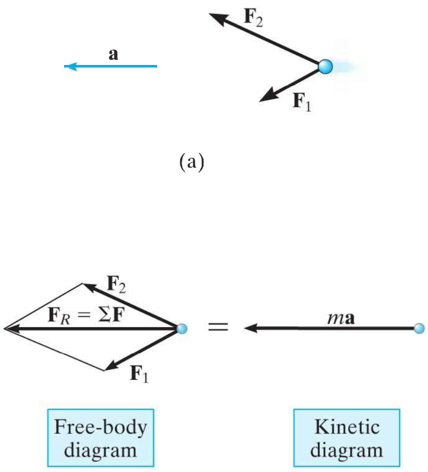


Fig. 13–2

13.2 The Equation of Motion

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $\mathbf{F}_R = \Sigma\mathbf{F}$. For this more general case, the equation of motion may be written as

$$\Sigma\mathbf{F} = m\mathbf{a} \qquad (13\text{–}4)$$

To illustrate application of this equation, consider the particle shown in Fig. 13–2*a*, which has a mass m and is subjected to the action of two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. We can graphically account for the magnitude and direction of each force acting on the particle by drawing the particle's *free-body diagram*, Fig. 13–2*b*. Since the *resultant* of these forces *produces* the vector $m\mathbf{a}$, its magnitude and direction can be represented graphically on the *kinetic diagram*, shown in Fig. 13–2*c*.* The equal sign written between the diagrams symbolizes the *graphical* equivalency between the free-body diagram and the kinetic diagram; i.e., $\Sigma\mathbf{F} = m\mathbf{a}$.† In particular, note that if $\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{0}$, then the acceleration is also zero, so that the particle will either remain at *rest* or move along a straight-line path with *constant velocity*. Such are the conditions of *static equilibrium*, Newton's first law of motion.

Inertial Reference Frame. When applying the equation of motion, it is important that the acceleration of the particle be measured with respect to a reference frame that is *either fixed or translates with a constant velocity*. In this way, the observer will not accelerate and measurements of the particle's acceleration will be the *same* from *any reference* of this type. Such a frame of reference is commonly known as a *Newtonian* or *inertial reference frame*, Fig. 13–3.

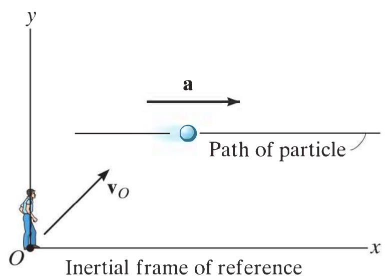


Fig. 13–3

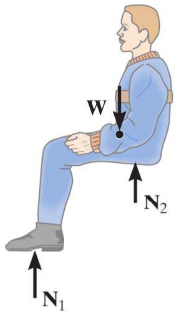
When studying the motions of rockets and satellites, it is justifiable to consider the inertial reference frame as fixed to the stars, whereas dynamics problems concerned with motions on or near the surface of the earth may be solved by using an inertial frame which is assumed fixed to the earth. Even though the earth both rotates about its own axis and revolves about the sun, the accelerations created by these rotations are relatively small and so they can be neglected for most applications.

*Recall the free-body diagram considers the particle to be free of its surrounding supports and shows all the forces acting on the particle. The kinetic diagram pertains to the particle's motion as caused by the forces.

†The equation of motion can also be rewritten in the form $\Sigma\mathbf{F} - m\mathbf{a} = \mathbf{0}$. The vector $-m\mathbf{a}$ is referred to as the *inertia force vector*. If it is treated in the same way as a "force vector," then the state of "equilibrium" created is referred to as *dynamic equilibrium*. This method of application is often referred to as the *D'Alembert principle*, named after the French mathematician Jean le Rond d'Alembert.

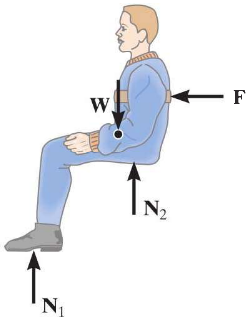
We are all familiar with the sensation one feels when sitting in a car that is subjected to a forward acceleration. Often people think this is caused by a "force" which acts on them and tends to push them back in their seats; however, this is not the case. Instead, this sensation occurs due to their inertia or the resistance of their mass to a change in velocity.

Consider the passenger who is strapped to the seat of a rocket sled. Provided the sled is at rest or is moving with constant velocity, then no force is exerted on his back as shown on his free-body diagram.



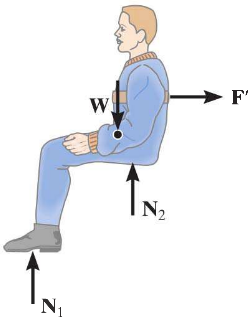
At rest or constant velocity

When the thrust of the rocket engine causes the sled to accelerate, then the seat upon which he is sitting exerts a force **F** on him which pushes him forward with the sled. In the photo, notice that the inertia of his head resists this change in motion (acceleration), and so his head moves back against the seat and his face, which is nonrigid, tends to distort backward.



Acceleration

Upon deceleration the force of the seatbelt **F′** tends to pull his body to a stop, but his head leaves contact with the back of the seat and his face distorts forward, again due to his inertia or tendency to continue to move forward. No force is pulling him forward, although this is the sensation he receives.



Deceleration

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13.3 Equation of Motion for a System of Particles

The equation of motion will now be extended to include a system of particles isolated within an enclosed region in space, as shown in Fig. 13–4*a*. In particular, there is no restriction in the way the particles are connected, so the following analysis applies equally well to the motion of a solid, liquid, or gas system.

At the instant considered, the arbitrary i-th particle, having a mass m_i, is subjected to a system of internal forces and a resultant external force. The *internal force*, represented symbolically as $\mathbf{f}_i$, is the resultant of all the forces the other particles exert on the ith particle. The *resultant external force* $\mathbf{F}_i$ represents, for example, the effect of gravitational, electrical, magnetic, or contact forces between the ith particle and adjacent bodies or particles *not* included within the system.

The free-body and kinetic diagrams for the ith particle are shown in Fig. 13–4*b*. Applying the equation of motion,

$$\Sigma \mathbf{F} = m\mathbf{a}; \qquad \mathbf{F}_i + \mathbf{f}_i = m_i\mathbf{a}_i$$

When the equation of motion is applied to each of the other particles of the system, similar equations will result. And, if all these equations are added together *vectorially*, we obtain

$$\Sigma \mathbf{F}_i + \Sigma \mathbf{f}_i = \Sigma m_i\mathbf{a}_i$$

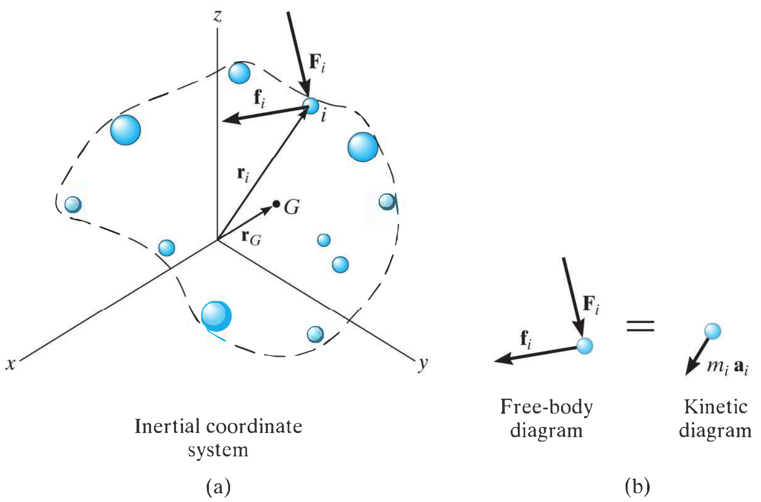

Fig. 13–4

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\Sigma \mathbf{F}_i = \Sigma m_i \mathbf{a}_i \qquad (13\text{–}5)$$

If $\mathbf{r}_G$ is a position vector which locates the *center of mass G* of the particles, Fig. 13–4*a*, then by definition of the center of mass, $m\mathbf{r}_G = \Sigma m_i \mathbf{r}_i$, where $m = \Sigma m_i$ is the total mass of all the particles. Differentiating this equation twice with respect to time, assuming that no mass is entering or leaving the system, yields

$$m\mathbf{a}_G = \Sigma m_i \mathbf{a}_i$$

Substituting this result into Eq. 13–5, we obtain

$$\boxed{\Sigma \mathbf{F} = m\mathbf{a}_G} \qquad (13\text{–}6)$$

Hence, the sum of the external forces acting on the system of particles is equal to the total mass of the particles times the acceleration of its center of mass G. Since in reality all particles must have a finite size to possess mass, Eq. 13–6 justifies application of the equation of motion to a *body* that is represented as a single particle.

Important Points

- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the *unbalanced force* on a particle causes it to *accelerate*.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

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13.4 Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial *x, y, z* frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their **i**, **j**, **k** components, Fig. 13–5. Applying the equation of motion, we have

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

For this equation to be satisfied, the respective **i**, **j**, **k** components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations:

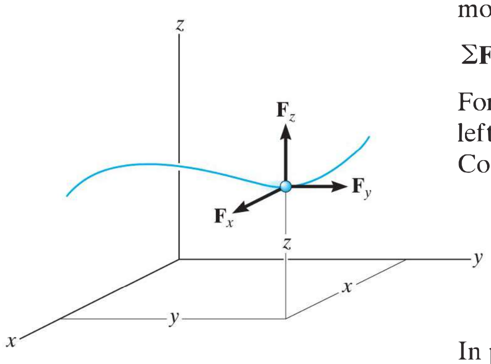

Fig. 13–5

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ \Sigma F_z &= ma_z \end{aligned} \qquad (13\text{–}7)$$

In particular, if the particle is constrained to move only in the *x–y* plane, then the first two of these equations are used to specify the motion.

Procedure for Analysis

The equations of motion are used to solve problems which require a relationship between the forces acting on a particle and the accelerated motion they cause.

Free-Body Diagram.

- Select the inertial coordinate system. Most often, rectangular or *x, y, z* coordinates are chosen to analyze problems for which the particle has *rectilinear motion.*
- Once the coordinates are established, draw the particle's free-body diagram. Drawing this diagram is *very important* since it provides a graphical representation that accounts for *all the forces* ($\Sigma \mathbf{F}$) which act on the particle, and thereby makes it possible to resolve these forces into their *x, y, z* components.
- The direction and sense of the particle's acceleration **a** should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the *same direction* as its *positive* inertial coordinate axis.
- The acceleration may be represented as the $m\mathbf{a}$ vector on the kinetic diagram.*
- Identify the unknowns in the problem.

*It is a convention in this text always to use the kinetic diagram as a graphical aid when developing the proofs and theory. The particle's acceleration or its components will be shown as blue colored vectors near the free-body diagram in the examples.

Equations of Motion.

- If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.
- If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.
- *Friction.* If a moving particle contacts a rough surface, it may be necessary to use the *frictional equation*, which relates the frictional and normal forces $\mathbf{F}_f$ and $\mathbf{N}$ acting at the surface of contact by using the coefficient of kinetic friction, i.e., $F_f = \mu_k N$. Remember that $\mathbf{F}_f$ always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts. If the particle is *on the verge* of relative motion, then the coefficient of static friction should be used.
- *Spring.* If the particle is connected to an *elastic spring* having negligible mass, the spring force F_s can be related to the deformation of the spring by the equation $F_s = ks$. Here k is the spring's stiffness measured as a force per unit length, and s is the stretch or compression defined as the difference between the deformed length l and the undeformed length l_0, i.e., $s = l - l_0$.

Kinematics.

- If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\Sigma\mathbf{F} = m\mathbf{a}$.
- If *acceleration* is a function of time, use $a = dv/dt$ and $v = ds/dt$ which, when integrated, yield the particle's velocity and position, respectively.
- If *acceleration* is a function of displacement, integrate $a\,ds = v\,dv$ to obtain the velocity as a function of position.
- If *acceleration is constant*, use $v = v_0 + a_c t$, $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$, $v^2 = v_0^2 + 2a_c(s - s_0)$ to determine the velocity or position of the particle.
- If the problem involves the dependent motion of several particles, use the method outlined in Sec. 12.9 to relate their accelerations. In all cases, make sure the positive inertial coordinate directions used for writing the kinematic equations are the same as those used for writing the equations of motion; otherwise, simultaneous solution of the equations will result in errors.
- If the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed.

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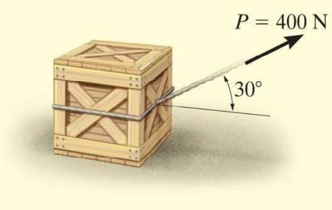
EXAMPLE 13.1



(a)

The 50-kg crate shown in Fig. 13–6*a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\text{ kg}\,(9.81\text{ m/s}^2) = 490.5\text{ N}$. As shown in Fig. 13–6*b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive *x* direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

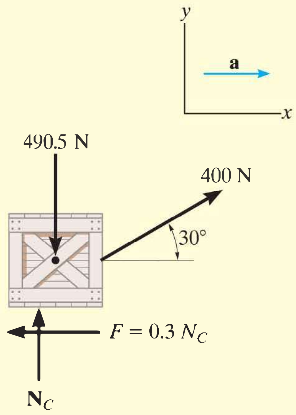
$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad 400\cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400\sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5\text{ N}$$

$$a = 5.185\text{ m/s}^2$$



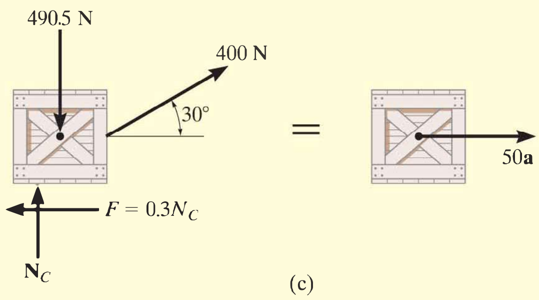
(b)

Fig. 13–6

Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$

$$= 15.6\text{ m/s} \rightarrow \qquad \textit{Ans.}$$



(c)

NOTE: We can also use the alternative procedure of drawing the crate's free-body *and* kinetic diagrams, Fig. 13–6*c*, prior to applying the equations of motion.

EXAMPLE 13.2

A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s, Fig. 13–7*a*. Determine the maximum height to which it will travel if (*a*) atmospheric resistance is neglected; and (*b*) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

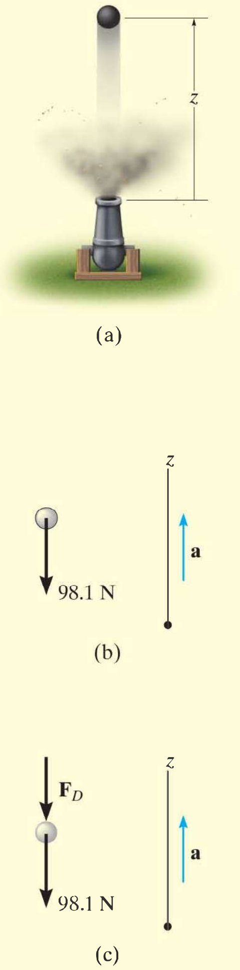


Fig. 13–7

SOLUTION

In both cases the known force on the projectile can be related to its acceleration using the equation of motion. Kinematics can then be used to relate the projectile's acceleration to its position.

Part (a) Free-Body Diagram. As shown in Fig. 13–7*b*, the projectile's weight is $W = mg = 10(9.81) = 98.1$ N. We will assume the unknown acceleration **a** acts upward in the *positive* z direction.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

The result indicates that the projectile, like every object having free-flight motion near the earth's surface, is subjected to a *constant* downward acceleration of 9.81 m/s².

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(z - z_0)$$

$$0 = (50)^2 + 2(-9.81)(h - 0)$$

$$h = 127 \text{ m} \qquad \textit{Ans.}$$

Part (b) Free-Body Diagram. Since the force $F_D = (0.01v^2)$ N tends to retard the upward motion of the projectile, it acts downward as shown on the free-body diagram, Fig. 13–7*c*.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -0.01v^2 - 98.1 = 10a, \qquad a = -(0.001v^2 + 9.81)$$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$$(+\uparrow)\ a\,dz = v\,dv; \qquad -(0.001v^2 + 9.81)\,dz = v\,dv$$

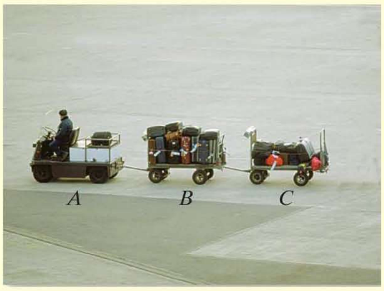
Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50}^{0} \frac{v\,dv}{0.001v^2 + 9.81} = -500 \ln(v^2 + 9810)\Big|_{50 \text{ m/s}}^{0}$$

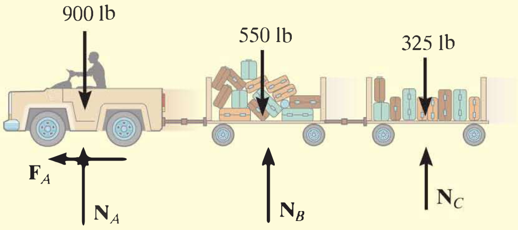
$$h = 114 \text{ m} \qquad \textit{Ans.}$$

NOTE: The answer indicates a lower elevation than that obtained in part (*a*) due to atmospheric resistance or drag.

EXAMPLE 13.3



The baggage truck A shown in the photo has a weight of 900 lb and tows a 550-lb cart B and a 325-lb cart C. For a short time the driving frictional force developed at the wheels of the truck is $F_A = (40t)$ lb, where t is in seconds. If the truck starts from rest, determine its speed in 2 seconds. Also, what is the horizontal force acting on the coupling between the truck and cart B at this instant? Neglect the size of the truck and carts.



(a)

SOLUTION

Free-Body Diagram. As shown in Fig. 13–8*a*, it is the frictional driving force that gives both the truck and carts an acceleration. Here we have considered all three vehicles as a single system.

Equation of Motion. Only motion in the horizontal direction has to be considered.

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 40t = \left(\frac{900 + 550 + 325}{32.2}\right)a$$

$$a = 0.7256t$$

Kinematics. Since the acceleration is a function of time, the velocity of the truck is obtained using $a = dv/dt$ with the initial condition that $v_0 = 0$ at $t = 0$. We have

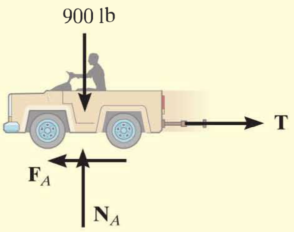
$$\int_0^v dv = \int_0^{2\,\text{s}} 0.7256t\,dt; \quad v = 0.3628t^2 \Big|_0^{2\,\text{s}} = 1.45 \text{ ft/s} \qquad \textit{Ans.}$$

Free-Body Diagram. In order to determine the force between the truck and cart B, we will consider a free-body diagram of the truck so that we can "expose" the coupling force **T** as external to the free-body diagram, Fig. 13–8*b*.

Equation of Motion. When $t = 2$ s, then

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 40(2) - T = \left(\frac{900}{32.2}\right)[0.7256(2)]$$

$$T = 39.4 \text{ lb} \qquad \textit{Ans.}$$



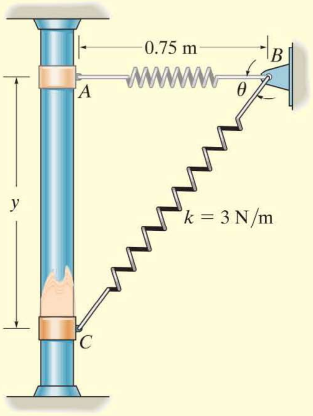
(b)

Fig. 13–8

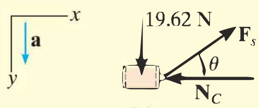
NOTE: Try and obtain this same result by considering a free-body diagram of carts B and C as a single system.

EXAMPLE 13.4

A smooth 2-kg collar C, shown in Fig. 13–9a, is attached to a spring having a stiffness $k = 3$ N/m and an unstretched length of 0.75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.



(a)



(b)

Fig. 13–9

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position y is shown in Fig. 13–9b. Furthermore, the collar is *assumed* to be accelerating so that "**a**" acts downward in the *positive* y direction. There are four unknowns, namely, N_C, F_s, a, and θ.

Equations of Motion.

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -N_C + F_s \cos\theta = 0 \qquad (1)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 19.62 - F_s \sin\theta = 2a \qquad (2)$$

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for N_C and a is possible once F_s and θ are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $F_s = ks$. Here the unstretched length is $AB = 0.75$ m, Fig. 13–9a; therefore, $s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75$. Since $k = 3$ N/m, then

$$F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right) \qquad (3)$$

From Fig. 13–9a, the angle θ is related to y by trigonometry.

$$\tan\theta = \frac{y}{0.75} \qquad (4)$$

Substituting $y = 1$ m into Eqs. 3 and 4 yields $F_s = 1.50$ N and $\theta = 53.1°$. Substituting these results into Eqs. 1 and 2, we obtain

$$N_C = 0.900 \text{ N} \qquad \textit{Ans.}$$

$$a = 9.21 \text{ m/s}^2 \downarrow \qquad \textit{Ans.}$$

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

EXAMPLE 13.5

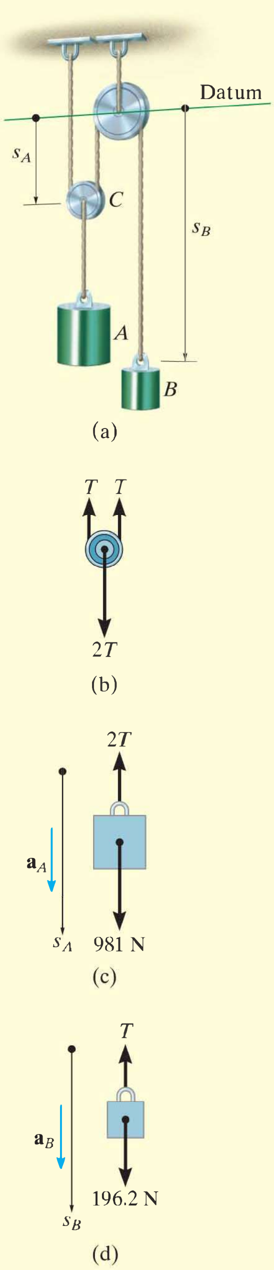


Fig. 13–10

The 100-kg block A shown in Fig. 13–10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the speed of the 20-kg block B in 2 s.

SOLUTION

Free-Body Diagrams. Since the mass of the pulleys is *neglected*, then for pulley C, $ma = 0$ and we can apply $\Sigma F_y = 0$ as shown in Fig. 13–10b. The free-body diagrams for blocks A and B are shown in Fig. 13–10c and d, respectively. Notice that for A to remain stationary $T = 490.5$ N, whereas for B to remain static $T = 196.2$ N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of $+s_A$ and $+s_B$. The three unknowns are T, a_A, and a_B.

Equations of Motion. Block A,

$$+\downarrow \Sigma F_y = ma_y; \qquad 981 - 2T = 100a_A \tag{1}$$

Block B,

$$+\downarrow \Sigma F_y = ma_y; \qquad 196.2 - T = 20a_B \tag{2}$$

Kinematics. The necessary third equation is obtained by relating a_A to a_B using a dependent motion analysis, discussed in Sect. 12.9. The coordinates s_A and s_B in Fig. 13–10a measure the positions of A and B from the fixed datum. It is seen that

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \tag{3}$$

Notice that when writing Eqs. 1 to 3, the *positive direction was always assumed downward*. It is very important to be *consistent* in this assumption since we are seeking a simultaneous solution of equations. The results are

$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

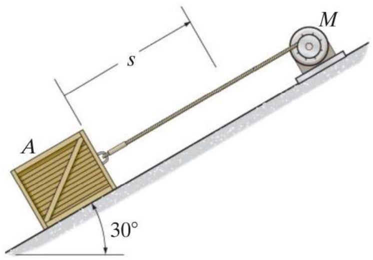
Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

$$(+\downarrow) \qquad v = v_0 + a_B t$$
$$= 0 + (-6.54)(2)$$
$$= -13.1 \text{ m/s} \qquad \textit{Ans.}$$

The negative sign indicates that block B is moving upward.

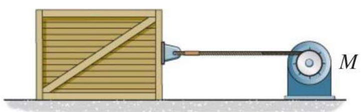
FUNDAMENTAL PROBLEMS

F13–1. The motor winds in the cable with a constant acceleration, such that the 20-kg crate moves a distance $s = 6$ m in 3 s, starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



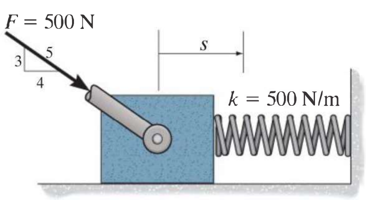
F13–1

F13–2. If motor M exerts a force of $F = (10t^2 + 100)$ N on the cable, where t is in seconds, determine the velocity of the 25-kg crate when $t = 4$ s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively. The crate is initially at rest.



F13–2

F13–3. A spring of stiffness $k = 500$ N/m is mounted against the 10-kg block. If the block is subjected to the force of $F = 500$ N, determine its velocity at $s = 0.5$ m. When $s = 0$, the block is at rest and the spring is uncompressed. The contact surface is smooth.

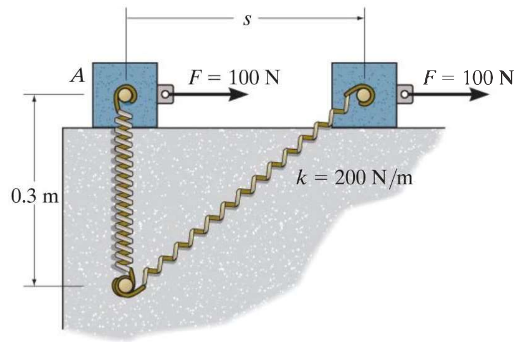


F13–3

F13–4. The 2-Mg car is being towed by a winch. If the winch exerts a force of $T = (100s)$ N on the cable, where s is the displacement of the car in meters, determine the speed of the car when $s = 10$ m, starting from rest. Neglect rolling resistance of the car.

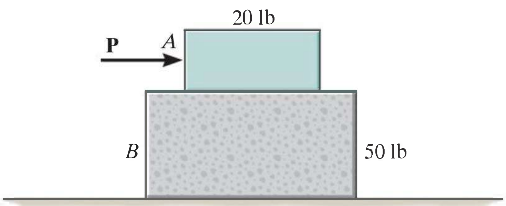
F13–4

F13–5. The spring has a stiffness $k = 200$ N/m and is unstretched when the 25-kg block is at A. Determine the acceleration of the block when $s = 0.4$ m. The contact surface between the block and the plane is smooth.



F13–5

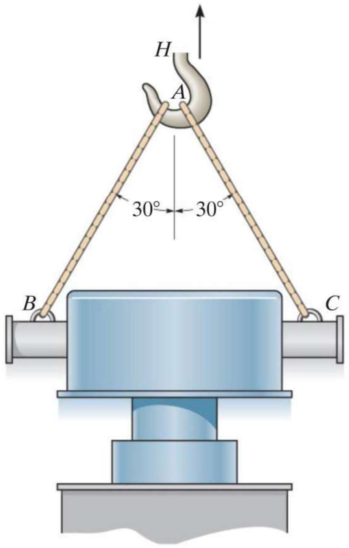
F13–6. Block B rests upon a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the acceleration of each block if $P = 6$ lb.



F13–6

PROBLEMS

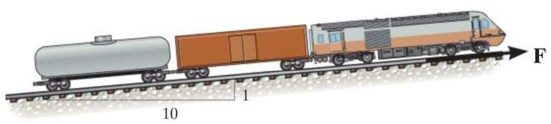
•**13–1.** The casting has a mass of 3 Mg. Suspended in a vertical position and initially at rest, it is given an upward speed of 200 mm/s in 0.3 s using a crane hook H. Determine the tension in cables AC and AB during this time interval if the acceleration is constant.



Prob. 13–1

13–2. The 160-Mg train travels with a speed of 80 km/h when it starts to climb the slope. If the engine exerts a traction force **F** of 1/20 of the weight of the train and the rolling resistance $\mathbf{F}_D$ is equal to 1/500 of the weight of the train, determine the deceleration of the train.

13–3. The 160-Mg train starts from rest and begins to climb the slope as shown. If the engine exerts a traction force **F** of 1/8 of the weight of the train, determine the speed of the train when it has traveled up the slope a distance of 1 km. Neglect rolling resistance.

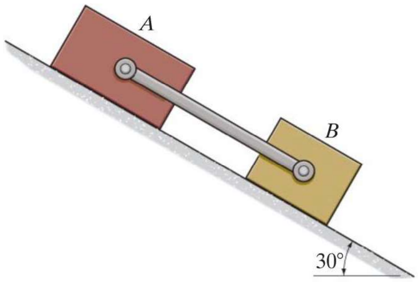


Probs. 13–2/3

*__13–4.__ The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.

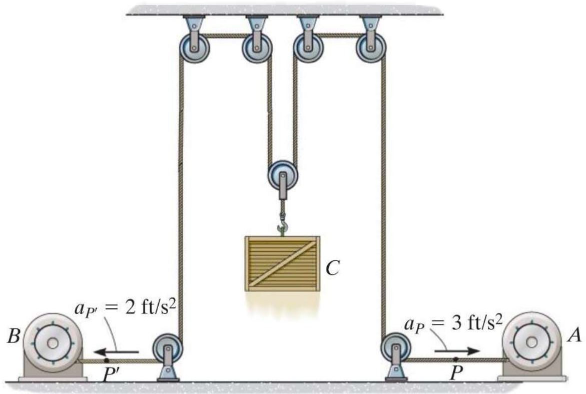
Prob. 13–4

•**13–5.** If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



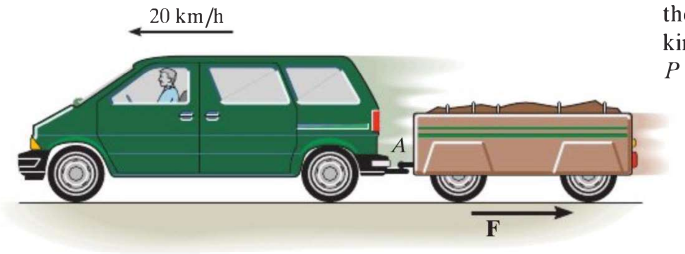
Prob. 13–5

13–6. Motors A and B draw in the cable with the accelerations shown. Determine the acceleration of the 300-lb crate C and the tension developed in the cable. Neglect the mass of all the pulleys.



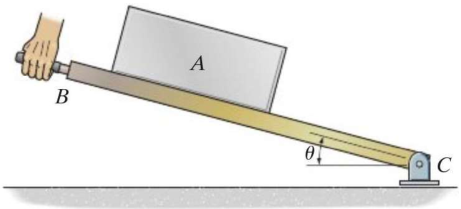
Prob. 13–6

13–7. The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



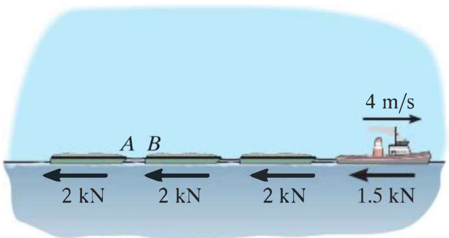
Prob. 13–7

***13–8.** If the 10-lb block A slides down the plane with a constant velocity when $\theta = 30°$, determine the acceleration of the block when $\theta = 45°$.



Prob. 13–8

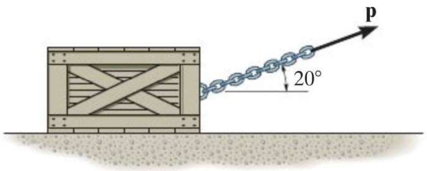
•13–9. Each of the three barges has a mass of 30 Mg, whereas the tugboat has a mass of 12 Mg. As the barges are being pulled forward with a constant velocity of 4 m/s, the tugboat must overcome the frictional resistance of the water, which is 2 kN for each barge and 1.5 kN for the tugboat. If the cable between A and B breaks, determine the acceleration of the tugboat.



Prob. 13–9

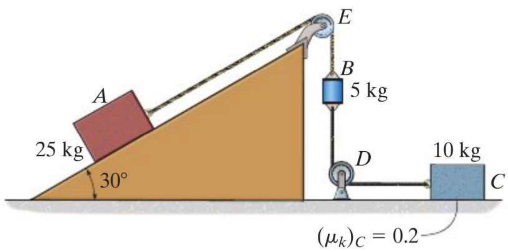
13–10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.

13–11. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



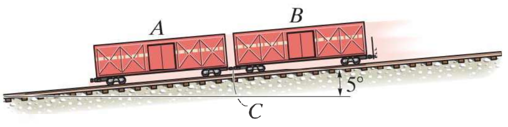
Probs. 13–10/11

***13–12.** Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $(\mu_k)_C = 0.2$.



Prob. 13–12

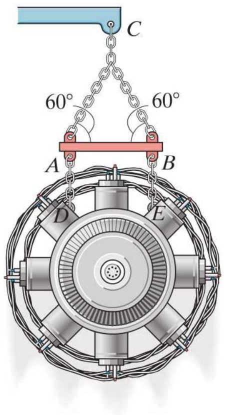
•13–13. The two boxcars A and B have a weight of 20 000 lb and 30 000 lb, respectively. If they coast freely down the incline when the brakes are applied to all the wheels of car A causing it to skid, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is $\mu_k = 0.5$. The wheels of car B are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on A and B, respectively.



Prob. 13–13

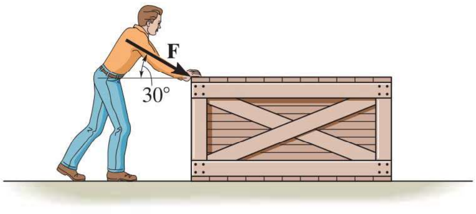
13–14. The 3.5-Mg engine is suspended from a spreader beam AB having a negligible mass and is hoisted by a crane which gives it an acceleration of $4\ \text{m/s}^2$ when it has a velocity of 2 m/s. Determine the force in chains CA and CB during the lift.

13–15. The 3.5-Mg engine is suspended from a spreader beam having a negligible mass and is hoisted by a crane which exerts a force of 40 kN on the hoisting cable. Determine the distance the engine is hoisted in 4 s, starting from rest.



Probs. 13–14/15

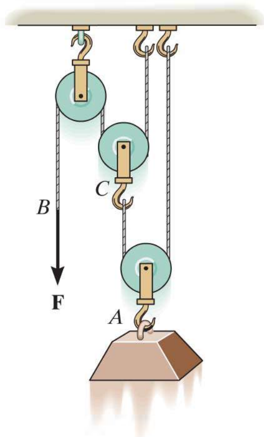
***13–16.** The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Prob. 13–16

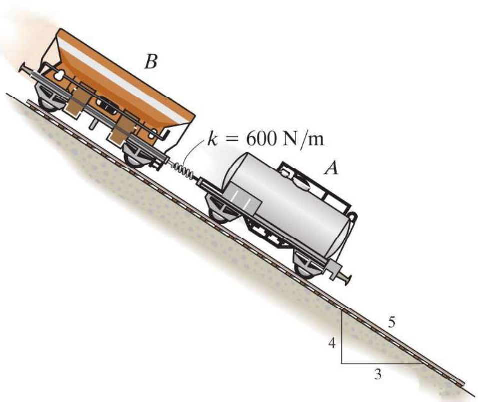
•13–17. A force of $F = 15$ lb is applied to the cord. Determine how high the 30-lb block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.

13–18. Determine the constant force **F** which must be applied to the cord in order to cause the 30-lb block A to have a speed of 12 ft/s when it has been displaced 3 ft upward starting from rest. Neglect the weight of the pulleys and cord.



Probs. 13–17/18

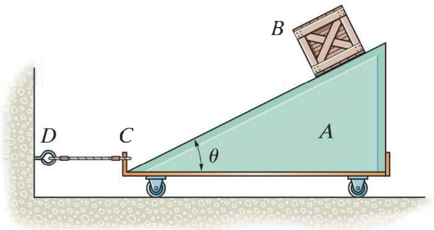
13–19. The 800-kg car at B is connected to the 350-kg car at A by a spring coupling. Determine the stretch in the spring if (a) the wheels of both cars are free to roll and (b) the brakes are applied to all four wheels of car B, causing the wheels to skid. Take $(\mu_k)_B = 0.4$. Neglect the mass of the wheels.



Prob. 13–19

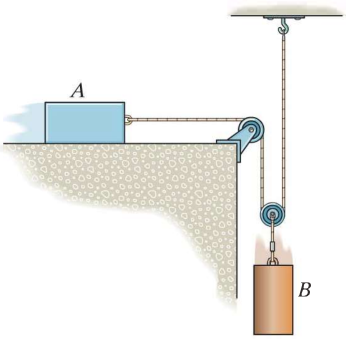
•13–21. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. Neglect friction.

13–22. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. The coefficient of kinetic friction between A and B is μ_k.



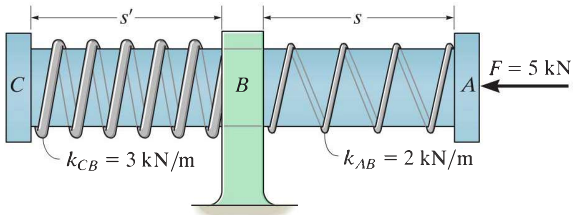
Probs. 13–21/22

***13–20.** The 10-lb block A travels to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.



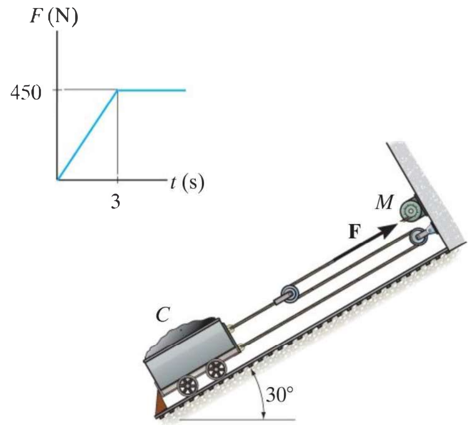
Prob. 13–20

13–23. The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



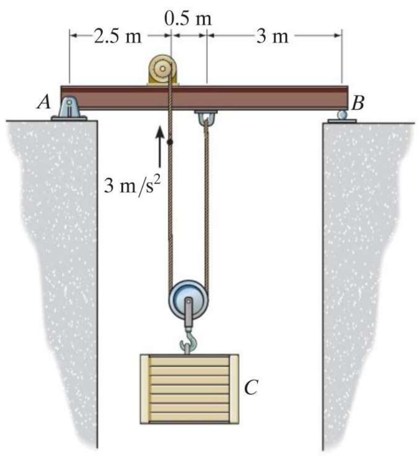
Prob. 13–23

***13–24.** If the force of the motor M on the cable is shown in the graph, determine the velocity of the cart when $t = 3$ s. The load and cart have a mass of 200 kg and the car starts from rest.



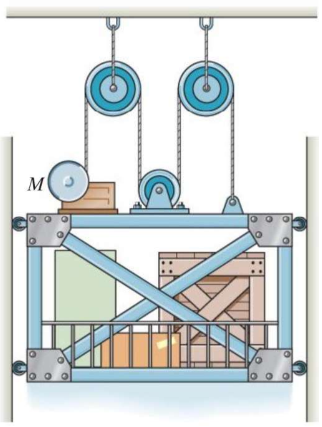
Prob. 13–24

•13–25. If the motor draws in the cable with an acceleration of $3\ \text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



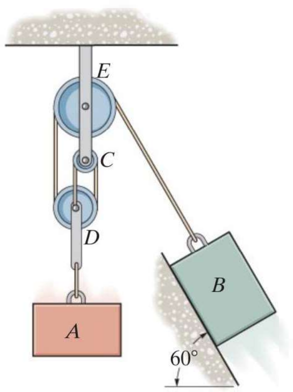
Prob. 13–25

13–26. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



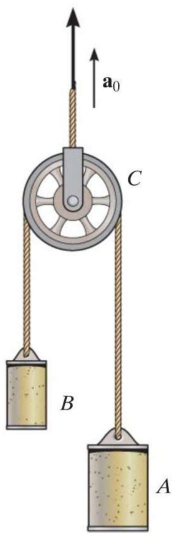
Prob. 13–26

13–27. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



Prob. 13–27

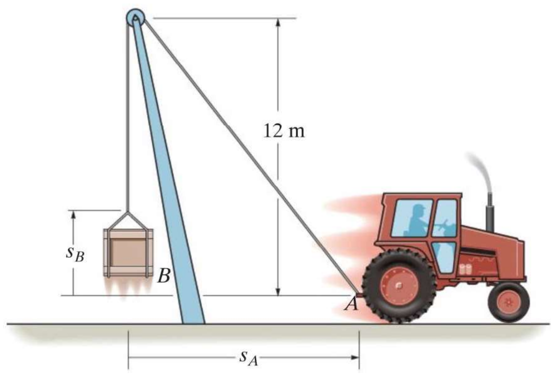
***13–28.** Blocks A and B have a mass of m_A and m_B, where $m_A > m_B$. If pulley C is given an acceleration of $\mathbf{a}_0$, determine the acceleration of the blocks. Neglect the mass of the pulley.



Prob. 13–28

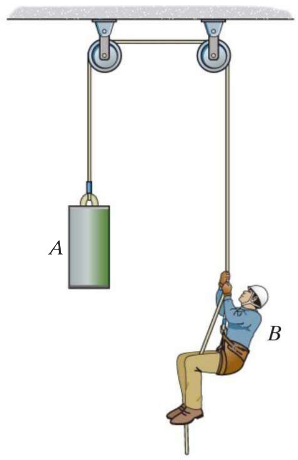
•13–29. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.

13–30. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



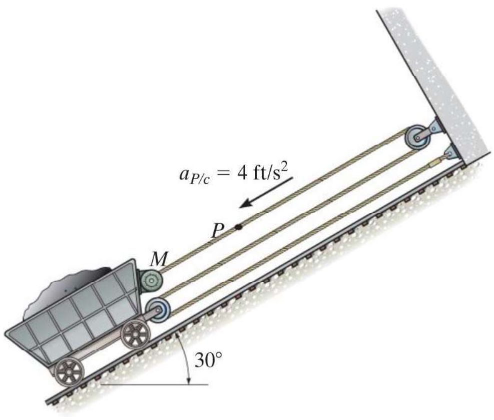
Probs. 13–29/30

13–31. The 75-kg man climbs up the rope with an acceleration of 0.25 m/s^2, measured relative to the rope. Determine the tension in the rope and the acceleration of the 80-kg block.



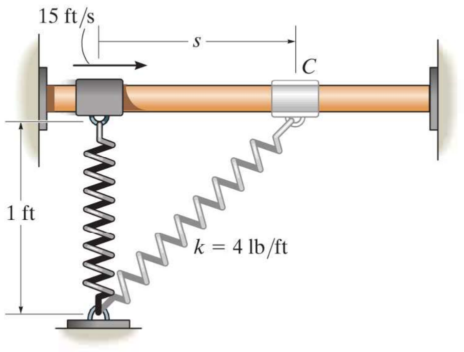
Prob. 13–31

***13–32.** Motor M draws in the cable with an acceleration of 4 ft/s^2, measured relative to the 200-lb mine car. Determine the acceleration of the car and the tension in the cable. Neglect the mass of the pulleys.



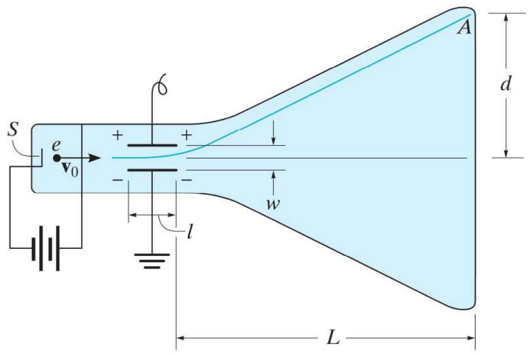
Prob. 13–32

•**13–33.** The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



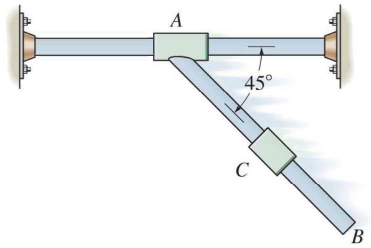
Prob. 13–33

13–34. In the cathode-ray tube, electrons having a mass m are emitted from a source point S and begin to travel horizontally with an initial velocity $\mathbf{v}_0$. While passing between the grid plates a distance l, they are subjected to a vertical force having a magnitude eV/w, where e is the charge of an electron, V the applied voltage acting across the plates, and w the distance between the plates. After passing clear of the plates, the electrons then travel in straight lines and strike the screen at A. Determine the deflection d of the electrons in terms of the dimensions of the voltage plate and tube. Neglect gravity which causes a slight vertical deflection when the electron travels from S to the screen, and the slight deflection between the plates.



Prob. 13–34

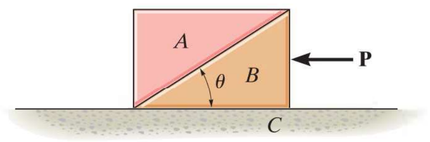
13–35. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves to the left at constant velocity along the horizontal guide, and (c) collar A is subjected to an acceleration of 2 m/s^2 to the left. In all cases, the motion occurs in the vertical plane.



Prob. 13–35

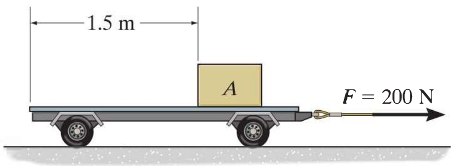
*__13–36.__ Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.

•**13–37.** Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.



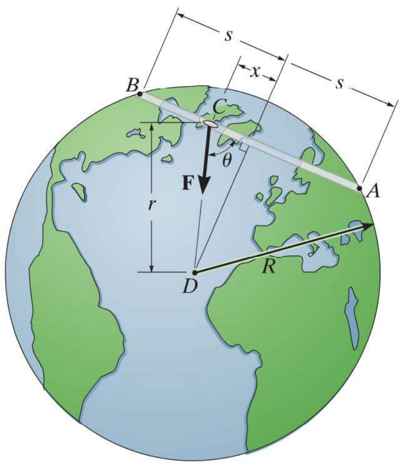
Probs. 13–36/37

13–38. If a force $F = 200$ N is applied to the 30-kg cart, show that the 20-kg block A will slide on the cart. Also determine the time for block A to move on the cart 1.5 m. The coefficients of static and kinetic friction between the block and the cart are $\mu_s = 0.3$ and $\mu_k = 0.25$. Both the cart and the block start from rest.



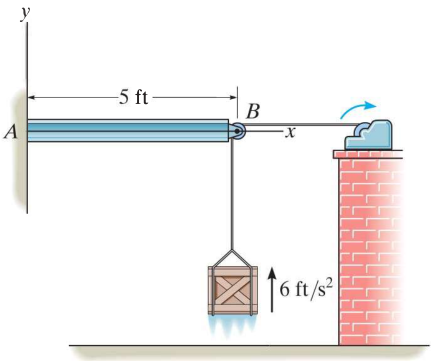
Prob. 13–38

13–39. Suppose it is possible to dig a smooth tunnel through the earth from a city at A to a city at B as shown. By the theory of gravitation, any vehicle C of mass m placed within the tunnel would be subjected to a gravitational force which is always directed toward the center of the earth D. This force **F** has a magnitude that is directly proportional to its distance r from the earth's center. Hence, if the vehicle has a weight of $W = mg$ when it is located on the earth's surface, then at an arbitrary location r the magnitude of force **F** is $F = (mg/R)r$, where $R = 6328$ km, the radius of the earth. If the vehicle is released from rest when it is at B, $x = s = 2$ Mm, determine the time needed for it to reach A, and the maximum velocity it attains. Neglect the effect of the earth's rotation in the calculation and assume the earth has a constant density. *Hint:* Write the equation of motion in the x direction, noting that $r \cos \theta = x$. Integrate, using the kinematic relation $v\,dv = a\,dx$, then integrate the result using $v = dx/dt$.



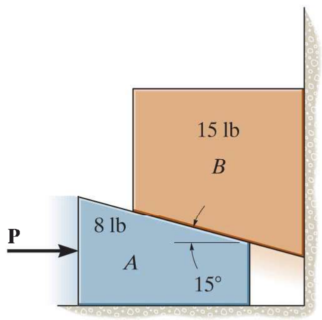
Prob. 13–39

***13–40.** The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at the fixed support A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



Prob. 13–40

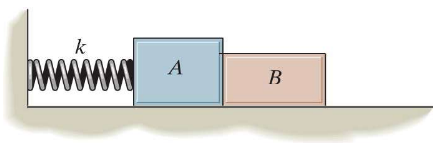
•**13–41.** If a horizontal force of $P = 10$ lb is applied to block A, determine the acceleration of block B. Neglect friction. *Hint:* Show that $a_B = a_A \tan 15°$.



Prob. 13–41

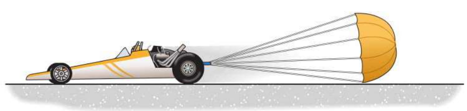
13–42. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?

13–43. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Probs. 13–42/43

***13–44.** The 600-kg dragster is traveling with a velocity of 125 m/s when the engine is shut off and the braking parachute is deployed. If air resistance imposed on the dragster due to the parachute is $F_D = (6000 + 0.9v^2)$ N, where v is in m/s, determine the time required for the dragster to come to rest.



Prob. 13–44

•13–45. The buoyancy force on the 500-kg balloon is $F = 6$ kN, and the air resistance is $F_D = (100v)$ N, where v is in m/s. Determine the terminal or maximum velocity of the balloon if it starts from rest.

Prob. 13–45

13–46. The parachutist of mass m is falling with a velocity of v_0 at the instant he opens the parachute. If air resistance is $F_D = Cv^2$, determine her maximum velocity (terminal velocity) during the descent.

Prob. 13–46

13–47. The weight of a particle varies with altitude such that $W = m(gr_0^2)/r^2$, where r_0 is the radius of the earth and r is the distance from the particle to the earth's center. If the particle is fired vertically with a velocity v_0 from the earth's surface, determine its velocity as a function of position r. What is the smallest velocity v_0 required to escape the earth's gravitational field, what is r_{max}, and what is the time required to reach this altitude?

13.5 Equations of Motion: Normal and Tangential Coordinates

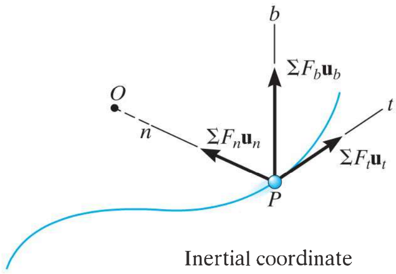
When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions, Fig. 13–11. Note that there is no motion of the particle in the binormal direction, since the particle is constrained to move along the path. We have

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$\Sigma F_t \mathbf{u}_t + \Sigma F_n \mathbf{u}_n + \Sigma F_b \mathbf{u}_b = m\mathbf{a}_t + m\mathbf{a}_n$$

This equation is satisfied provided

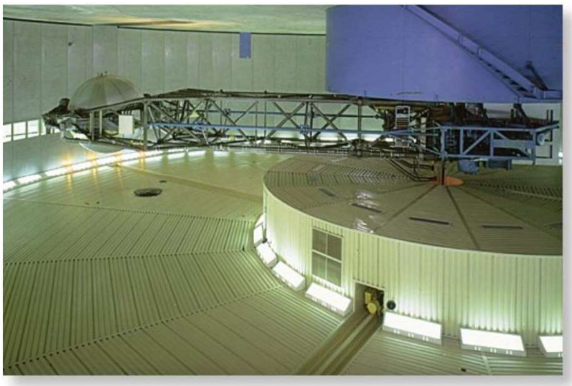
$$\Sigma F_t = ma_t$$
$$\Sigma F_n = ma_n \quad (13\text{–}8)$$
$$\Sigma F_b = 0$$



Inertial coordinate system

Fig. 13–11

Recall that $a_t\,(= dv/dt)$ represents the time rate of change in the magnitude of velocity. So if $\Sigma \mathbf{F}_t$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, $a_n\,(= v^2/\rho)$ represents the time rate of change in the velocity's direction. It is caused by $\Sigma \mathbf{F}_n$, which *always* acts in the positive n direction, i.e., toward the path's center of curvature. From this reason it is often referred to as the *centripetal force*.



The centrifuge is used to subject a passenger to a very large normal acceleration caused by rapid rotation. Realize that this acceleration is *caused by* the unbalanced normal force exerted on the passenger by the seat of the centrifuge.

Procedure for Analysis

When a problem involves the motion of a particle along a *known curved path*, normal and tangential coordinates should be considered for the analysis since the acceleration components can be readily formulated. The method for applying the equations of motion, which relate the forces to the acceleration, has been outlined in the procedure given in Sec. 13.4. Specifically, for *t, n, b* coordinates it may be stated as follows:

Free-Body Diagram.

- Establish the inertial *t, n, b* coordinate system at the particle and draw the particle's free-body diagram.
- The particle's normal acceleration $\mathbf{a}_n$ *always* acts in the positive n direction.
- If the tangential acceleration $\mathbf{a}_t$ is unknown, assume it acts in the positive t direction.
- There is no acceleration in the b direction.
- Identify the unknowns in the problem.

Equations of Motion.

- Apply the equations of motion, Eqs. 13–8.

Kinematics.

- Formulate the tangential and normal components of acceleration; i.e., $a_t = dv/dt$ or $a_t = v\,dv/ds$ and $a_n = v^2/\rho$.
- If the path is defined as $y = f(x)$, the radius of curvature at the point where the particle is located can be obtained from $\rho = [1 + (dy/dx)^2]^{3/2}/|d^2y/dx^2|$.

EXAMPLE 13.6

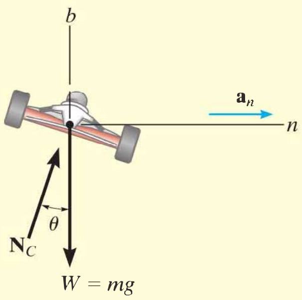
Determine the banking angle θ for the race track so that the wheels of the racing cars shown in Fig. 13–12*a* will not have to depend upon friction to prevent any car from sliding up or down the track. Assume the cars have negligible size, a mass m, and travel around the curve of radius ρ with a constant speed v.

(a)

SOLUTION

Before looking at the following solution, give some thought as to why it should be solved using t, n, b coordinates.

Free-Body Diagram. As shown in Fig. 13–12*b*, and as stated in the problem, no frictional force acts on the car. Here $\mathbf{N}_C$ represents the *resultant* of the ground on all four wheels. Since a_n can be calculated, the unknowns are N_C and θ.



(b)

Fig. 13–12

Equations of Motion. Using the n, b axes shown,

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad N_C \sin\theta = m\frac{v^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_b = 0; \qquad N_C \cos\theta - mg = 0 \qquad (2)$$

Eliminating N_C and m from these equations by dividing Eq. 1 by Eq. 2, we obtain

$$\tan\theta = \frac{v^2}{g\rho}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{g\rho}\right) \qquad \textit{Ans.}$$

NOTE: The result is independent of the mass of the car. Also, a force summation in the tangential direction is of no consequence to the solution. If it were considered, then $a_t = dv/dt = 0$, since the car moves with *constant speed*. A further analysis of this problem is discussed in Prob. 21–47.

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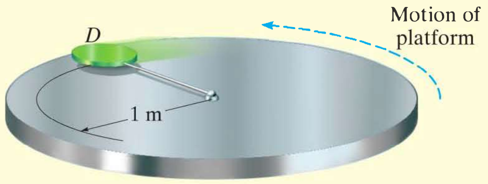
EXAMPLE 13.7

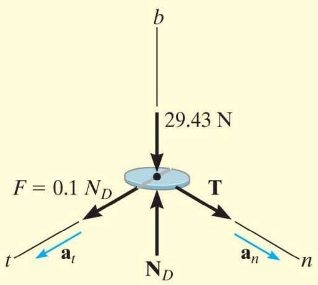
The 3-kg disk D is attached to the end of a cord as shown in Fig. 13–13a. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



(a)

SOLUTION

Free-Body Diagram. The frictional force has a magnitude $F = \mu_k N_D = 0.1N_D$ and a sense of direction that opposes the *relative motion* of the disk with respect to the platform. It is this force that gives the disk a tangential component of acceleration causing v to increase, thereby causing T to increase until it reaches 100 N. The weight of the disk is $W = 3(9.81) = 29.43$ N. Since a_n can be related to v, the unknowns are N_D, a_t, and v.



(b)

Fig. 13–13

Equations of Motion.

$$\Sigma F_n = ma_n; \qquad T = 3\left(\frac{v^2}{1}\right) \qquad (1)$$

$$\Sigma F_t = ma_t; \qquad 0.1N_D = 3a_t \qquad (2)$$

$$\Sigma F_b = 0; \qquad N_D - 29.43 = 0 \qquad (3)$$

Setting $T = 100$ N, Eq. 1 can be solved for the critical speed v_{cr} of the disk needed to break the cord. Solving all the equations, we obtain

$$N_D = 29.43 \text{ N}$$
$$a_t = 0.981 \text{ m/s}^2$$
$$v_{cr} = 5.77 \text{ m/s}$$

Kinematics. Since a_t is *constant*, the time needed to break the cord is

$$v_{cr} = v_0 + a_t t$$
$$5.77 = 0 + (0.981)t$$
$$t = 5.89 \text{ s} \qquad \textit{Ans.}$$

EXAMPLE 13.8

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown in Fig. 13–14*a*, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point A, where her velocity is 65 ft/s. Also, what is her acceleration at this point?

SOLUTION

Why consider using n, t coordinates to solve this problem?

Free-Body Diagram. Since $dy/dx = x/100\,|_{x=0} = 0$, the slope at A is horizontal. The free-body diagram of the skier when she is at A is shown in Fig. 13–14*b*. Since the path is *curved*, there are two components of acceleration, $\mathbf{a}_n$ and $\mathbf{a}_t$. Since a_n can be calculated, the unknowns are a_t and N_A.

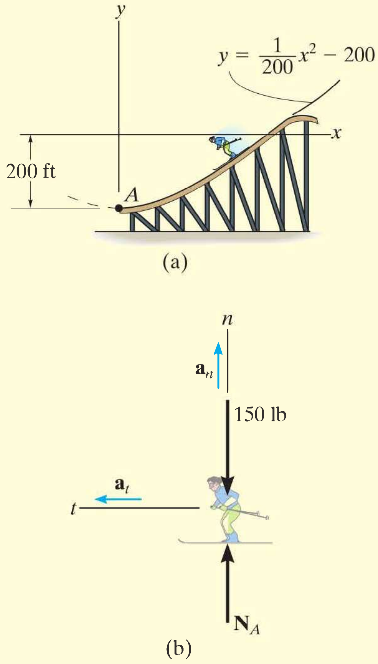


Fig. 13–14

Equations of Motion.

$$+\uparrow \Sigma F_n = ma_n; \qquad N_A - 150 = \frac{150}{32.2}\left(\frac{(65)^2}{\rho}\right) \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_t = ma_t; \qquad 0 = \frac{150}{32.2}a_t \qquad (2)$$

The radius of curvature ρ for the path must be determined at point $A(0, -200\text{ ft})$. Here $y = \frac{1}{200}x^2 - 200$, $dy/dx = \frac{1}{100}x$, $d^2y/dx^2 = \frac{1}{100}$, so that at $x = 0$,

$$\rho = \left.\frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}\right|_{x=0} = \frac{[1 + (0)^2]^{3/2}}{|\frac{1}{100}|} = 100\text{ ft}$$

Substituting this into Eq. 1 and solving for N_A, we obtain

$$N_A = 347\text{ lb} \qquad \textit{Ans.}$$

Kinematics. From Eq. 2,

$$a_t = 0$$

Thus,

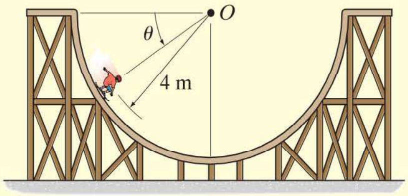
$$a_n = \frac{v^2}{\rho} = \frac{(65)^2}{100} = 42.2\text{ ft/s}^2$$

$$a_A = a_n = 42.2\text{ ft/s}^2 \uparrow \qquad \textit{Ans.}$$

NOTE: Apply the equation of motion in the y direction and show that when the skier is in midair her acceleration is 32.2 ft/s^2.

13

EXAMPLE 13.9

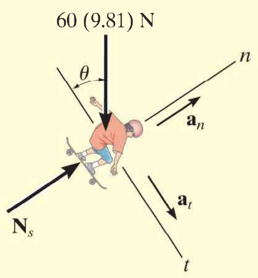


(a)

The 60-kg skateboarder in Fig.13–15*a* coasts down the circular track. If he starts from rest when $\theta = 0°$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60°$. Neglect his size for the calculation.

SOLUTION

Free-Body Diagram. The free-body diagram of the skateboarder when he is at an *arbitrary position* θ is shown in Fig. 13–15*b*. At $\theta = 60°$ there are three unknowns, N_s, a_t, and a_n (or v).



(b)

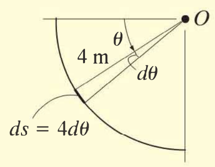
Equations of Motion.

$$\swarrow \Sigma F_n = ma_n; \quad N_S - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{m}}\right) \quad (1)$$

$$\searrow \Sigma F_t = ma_t; \quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\, a_t$$

$$a_t = 9.81\cos\theta$$

Kinematics. Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60°$. Using the geometric relation $s = \theta r$, where $ds = r\,d\theta = (4\text{ m})\,d\theta$, Fig. 13–15*c*, and the initial condition $v = 0$ at $\theta = 0°$, we have,



(c)

Fig. 13–15

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60°} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60°}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60° - 0)$$

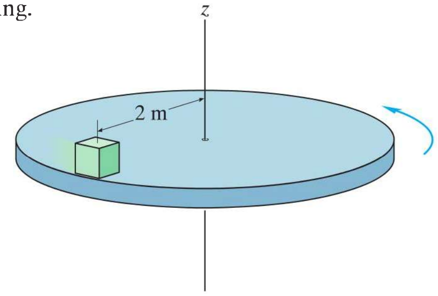
$$v^2 = 67.97\text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60°$ into Eq. (1), yields

$$N_s = 1529.23\text{ N} = 1.53\text{ kN} \qquad \textit{Ans.}$$

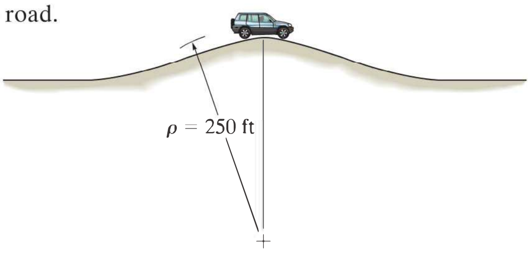
FUNDAMENTAL PROBLEMS

F13–7. The block rests at a distance of 2 m from the center of the platform. If the coefficient of static friction between the block and the platform is $\mu_s = 0.3$, determine the maximum speed which the block can attain before it begins to slip. Assume the angular motion of the disk is slowly increasing.



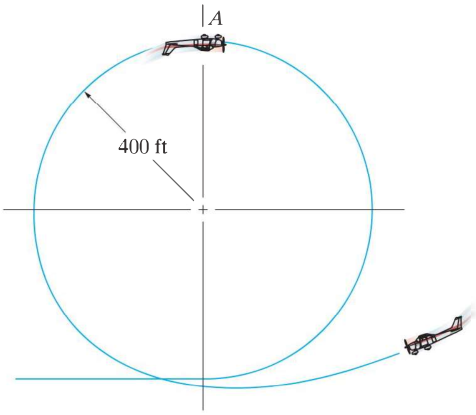
F13–7

F13–8. Determine the maximum speed that the jeep can travel over the crest of the hill and not lose contact with the road.



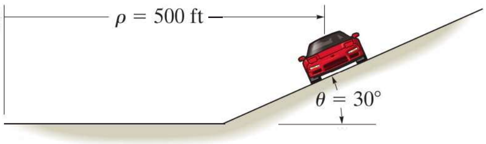
F13–8

F13–9. A pilot weighs 150 lb and is traveling at a constant speed of 120 ft/s. Determine the normal force he exerts on the seat of the plane when he is upside down at A. The loop has a radius of curvature of 400 ft.



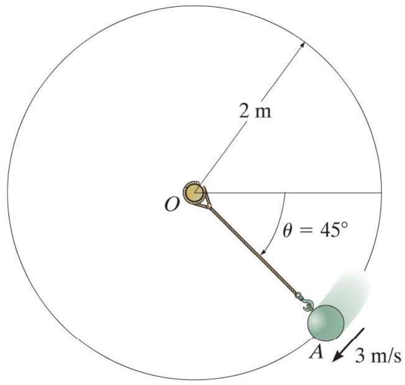
F13–9

F13–10. The sports car is traveling along a 30° banked road having a radius of curvature of $\rho = 500$ ft. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the maximum safe speed so no slipping occurs. Neglect the size of the car.



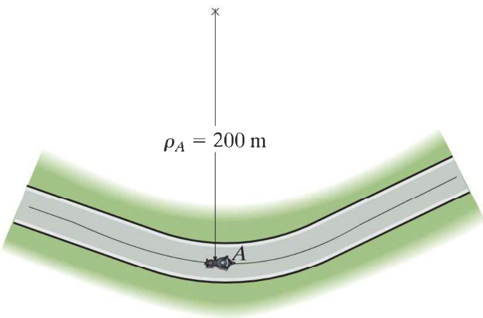
F13–10

F13–11. If the 10-kg ball has a velocity of 3 m/s when it is at the position A, along the vertical path, determine the tension in the cord and the increase in the speed of the ball at this position.



F13–11

F13–12. The motorcycle has a mass of 0.5 Mg and a negligible size. It passes point A traveling with a speed of 15 m/s, which is increasing at a constant rate of 1.5 m/s^2. Determine the resultant frictional forcce exerted by the road on the tires at this instant.

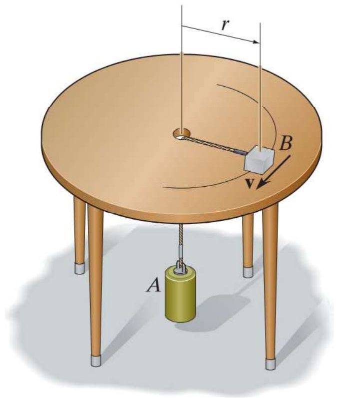


F13–12

PROBLEMS

***13–48.** The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\text{ m/s}$, determine the radius r of the circular path along which it travels.

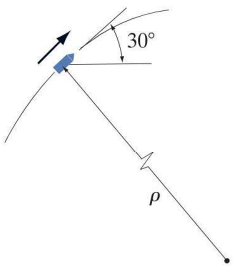
•13–49. The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\text{ m}$, determine the speed of the block.



Probs. 13–48/49

13–50. At the instant shown, the 50-kg projectile travels in the vertical plane with a speed of $v = 40\text{ m/s}$. Determine the tangential component of its acceleration and the radius of curvature ρ of its trajectory at this instant.

13–51. At the instant shown, the radius of curvature of the vertical trajectory of the 50-kg projectile is $\rho = 200\text{ m}$. Determine the speed of the projectile at this instant.

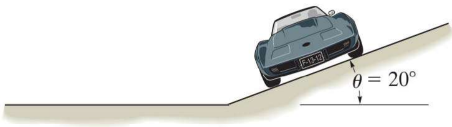


Probs. 13–50/51

***13–52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is $149.6(10^6)$ km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

•13–53. The sports car, having a mass of 1700 kg, travels horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.

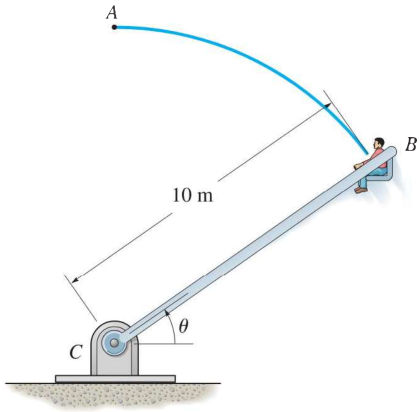
13–54. Using the data in Prob. 13–53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.



Probs. 13–53/54

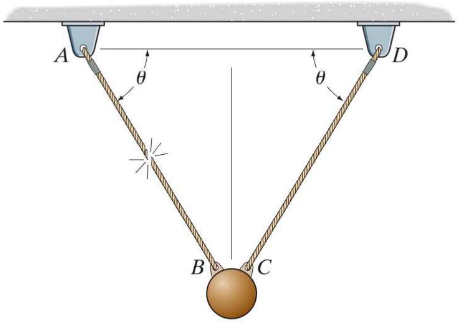
13–55. The device shown is used to produce the experience of weightlessness in a passenger when he reaches point A, $\theta = 90°$, along the path. If the passenger has a mass of 75 kg, determine the minimum speed he should have when he reaches A so that he does not exert a normal reaction on the seat. The chair is pin-connected to the frame BC so that he is always seated in an upright position. During the motion his speed remains constant.

***13–56.** A man having the mass of 75 kg sits in the chair which is pin-connected to the frame BC. If the man is always seated in an upright position, determine the horizontal and vertical reactions of the chair on the man at the instant $\theta = 45°$. At this instant he has a speed of 6 m/s, which is increasing at 0.5 m/s^2.



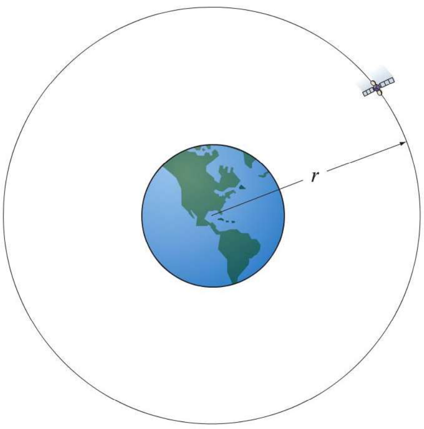
Probs. 13–55/56

•13–57. Determine the tension in wire CD just after wire AB is cut. The small bob has a mass m.



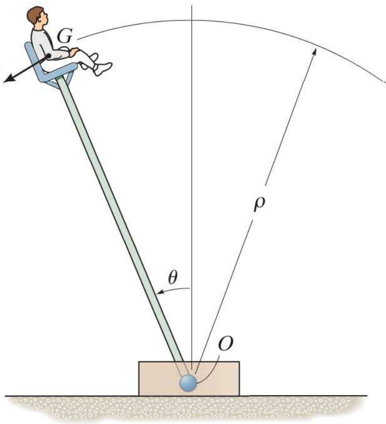
Prob. 13–57

13–58. Determine the time for the satellite to complete its orbit around the earth. The orbit has a radius r measured from the center of the earth. The masses of the satellite and the earth are m_s and M_e, respectively.



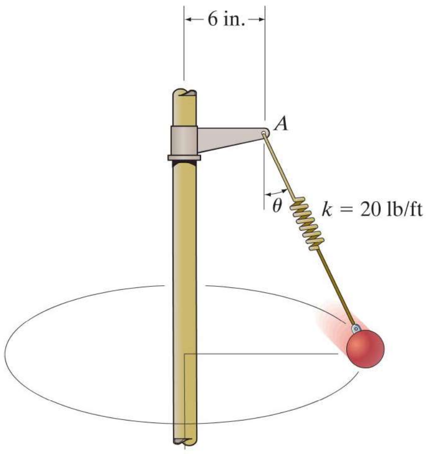
Prob. 13–58

13–59. An acrobat has a weight of 150 lb and is sitting on a chair which is perched on top of a pole as shown. If by a mechanical drive the pole rotates downward at a constant rate from $\theta = 0°$, such that the acrobat's center of mass G maintains a *constant speed* of $v_a = 10$ ft/s, determine the angle θ at which he begins to "fly" out of the chair. Neglect friction and assume that the distance from the pivot O to G is $\rho = 15$ ft.



Prob. 13–59

***13–60.** A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path.

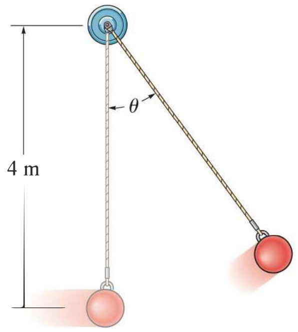


Prob. 13–60

13

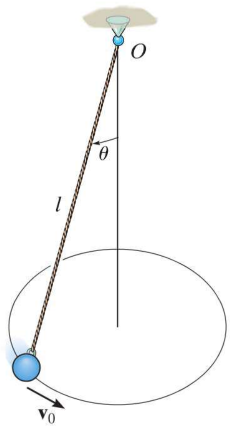
•**13–61.** If the ball has a mass of 30 kg and a speed $v = 4\text{ m/s}$ at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings and momentarily stops. Neglect the size of the ball.

13–62. The ball has a mass of 30 kg and a speed $v = 4\text{ m/s}$ at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



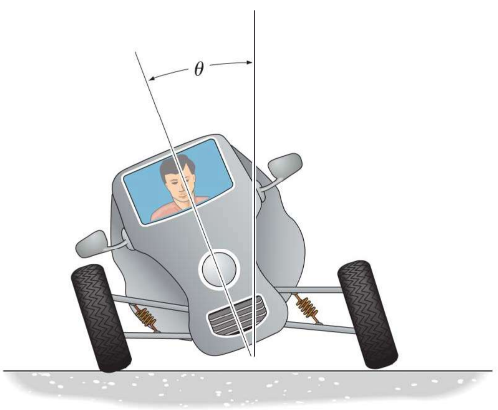
Probs. 13–61/62

*__13–64.__ The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta \sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



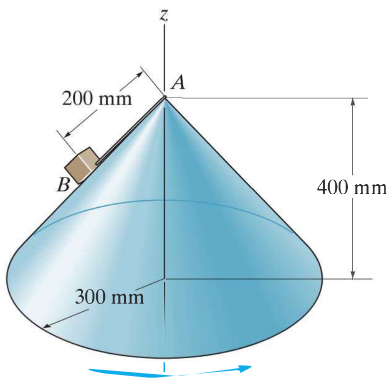
Prob. 13–64

13–63. The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



Prob. 13–63

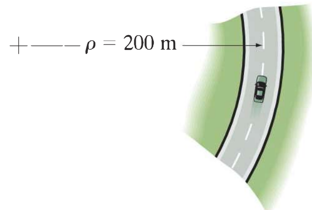
•**13–65.** The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. If the block has a speed of 0.5 m/s around the cone, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



Prob. 13–65

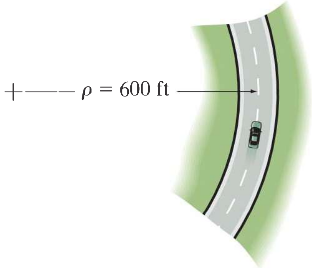
13–66. Determine the minimum coefficient of static friction between the tires and the road surface so that the 1.5-Mg car does not slide as it travels at 80 km/h on the curved road. Neglect the size of the car.

13–67. If the coefficient of static friction between the tires and the road surface is $\mu_s = 0.25$, determine the maximum speed of the 1.5-Mg car without causing it to slide when it travels on the curve. Neglect the size of the car.



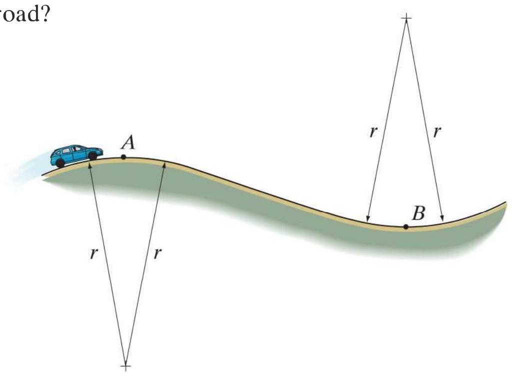
Probs. 13–66/67

***13–68.** At the instant shown, the 3000-lb car is traveling with a speed of 75 ft/s, which is increasing at a rate of $6\,\text{ft/s}^2$. Determine the magnitude of the resultant frictional force the road exerts on the tires of the car. Neglect the size of the car.



Prob. 13–68

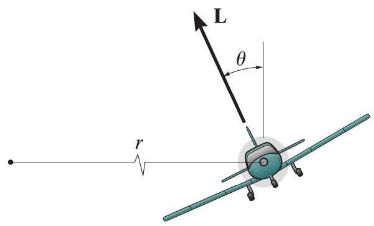
•13–69. Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



Prob. 13–69

13–70. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000\,\text{m}$. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.

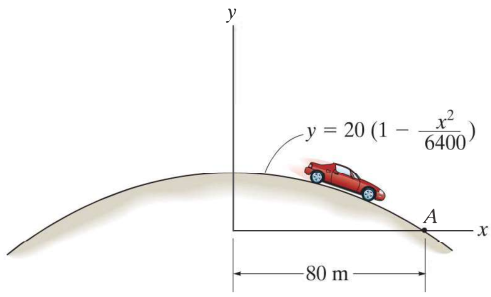
13–71. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15°$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



Probs. 13–70/71

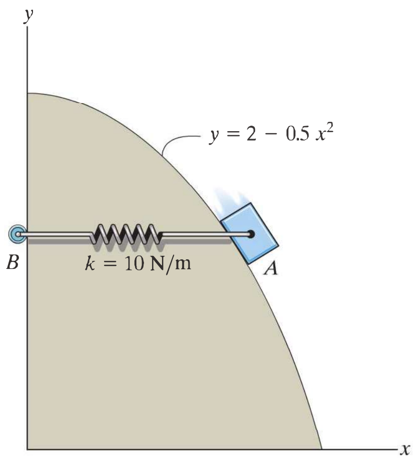
***13–72.** The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

•13–73. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at $3\ \text{m/s}^2$. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



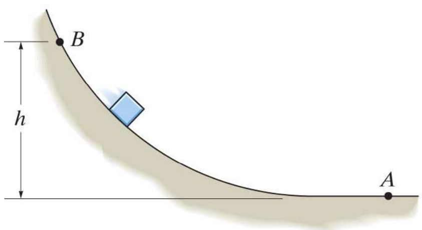
Probs. 13–72/73

13–74. The 6-kg block is confined to move along the smooth parabolic path. The attached spring restricts the motion and, due to the roller guide, always remains horizontal as the block descends. If the spring has a stiffness of $k = 10$ N/m, and unstretched length of 0.5 m, determine the normal force of the path on the block at the instant $x = 1$ m when the block has a speed of 4 m/s. Also, what is the rate of increase in speed of the block at this point? Neglect the mass of the roller and the spring.



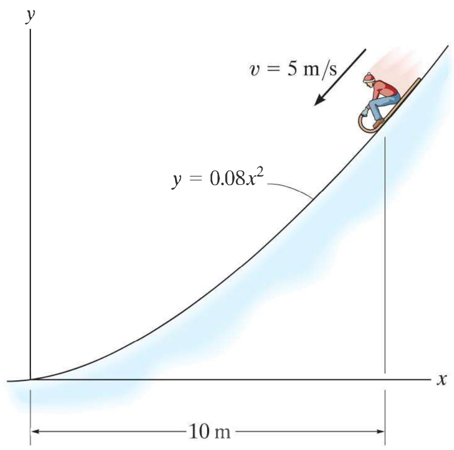
Prob. 13–74

13–75. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



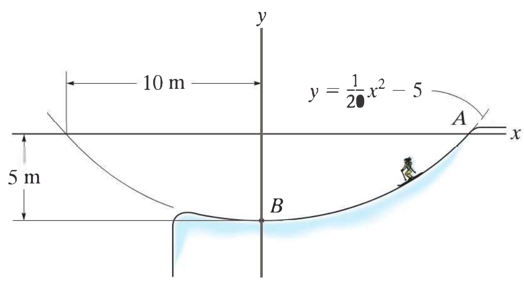
Prob. 13–75

***13–76.** A toboggan and rider of total mass 90 kg travel down along the (smooth) slope defined by the equation $y = 0.08x^2$. At the instant $x = 10$ m, the toboggan's speed is 5 m/s. At this point, determine the rate of increase in speed and the normal force which the slope exerts on the toboggan. Neglect the size of the toboggan and rider for the calculation.



Prob. 13–76

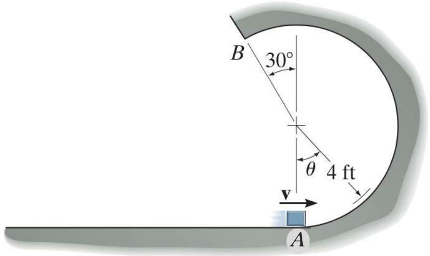
•13–77. The skier starts from rest at $A(10 \text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–75.



Prob. 13–77

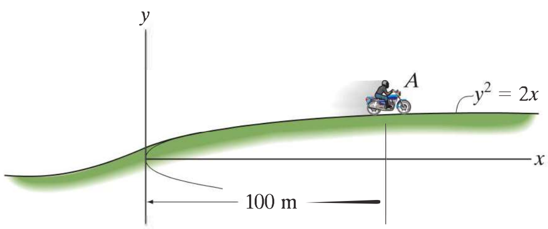
13–78. The 5-lb box is projected with a speed of 20 ft/s at A up the vertical circular smooth track. Determine the angle θ when the box leaves the track.

13–79. Determine the minimum speed that must be given to the 5-lb box at A in order for it to remain in contact with the circular path. Also, determine the speed of the box when it reaches point B.



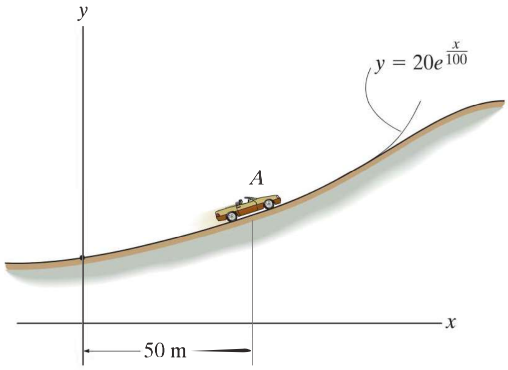
Probs. 13–78/79

***13–80.** The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



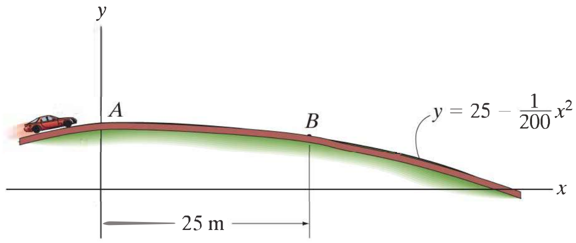
Prob. 13–80

•13–81. The 1.8-Mg car travels up the incline at a constant speed of 80 km/h. Determine the normal reaction of the road on the car when it reaches point A. Neglect its size.



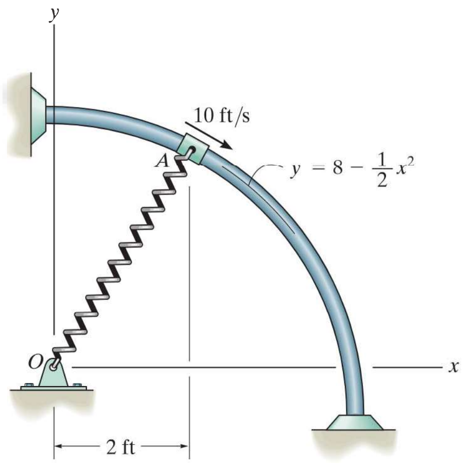
Prob. 13–81

13–82. Determine the maximum speed the 1.5-Mg car can have and still remain in contact with the road when it passes point A. If the car maintains this speed, what is the normal reaction of the road on it when it passes point B? Neglect the size of the car.

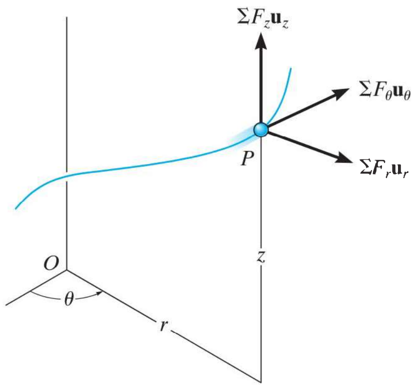


Prob. 13–82

13–83. The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



Prob. 13–83



Inertial coordinate system

Fig. 13–16

13.6 Equations of Motion: Cylindrical Coordinates

When all the forces acting on a particle are resolved into cylindrical components, i.e., along the unit-vector directions $\mathbf{u}_r$, $\mathbf{u}_\theta$, $\mathbf{u}_z$, Fig. 13–16, the equation of motion can be expressed as

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$\Sigma F_r \mathbf{u}_r + \Sigma F_\theta \mathbf{u}_\theta + \Sigma F_z \mathbf{u}_z = ma_r \mathbf{u}_r + ma_\theta \mathbf{u}_\theta + ma_z \mathbf{u}_z$$

To satisfy this equation, we require

$$\Sigma F_r = ma_r$$
$$\Sigma F_\theta = ma_\theta \qquad (13\text{–}9)$$
$$\Sigma F_z = ma_z$$

If the particle is constrained to move only in the r–θ plane, then only the first two of Eqs. 13–9 are used to specify the motion.

As the car of weight W descends the spiral track, the resultant normal force which the track exerts on the car can be represented by its three cylindrical components. $-\mathbf{N}_r$ creates a radial acceleration, $-\mathbf{a}_r$, $\mathbf{N}_\theta$ creates a transverse acceleration $\mathbf{a}_\theta$, and the difference $\mathbf{W} - \mathbf{N}_z$ creates an azimuthal acceleration $-\mathbf{a}_z$.

Tangential and Normal Forces. The most straightforward type of problem involving cylindrical coordinates requires the determination of the resultant force components ΣF_r, ΣF_θ, ΣF_z which cause a particle to move with a *known* acceleration. If, however, the particle's accelerated motion is not completely specified at the given instant, then some information regarding the directions or magnitudes of the forces acting on the particle must be known or computed in order to solve Eqs. 13–9. For example, the force $\mathbf{P}$ causes the particle in Fig. 13–17*a* to move along a path $r = f(\theta)$. The *normal force* $\mathbf{N}$ which the path exerts on the particle is always *perpendicular to the tangent of the path*, whereas the frictional force $\mathbf{F}$ always acts along the tangent in the opposite direction of motion. The *directions* of $\mathbf{N}$ and $\mathbf{F}$ can be specified relative to the radial coordinate by using the angle ψ (psi), Fig. 13–17*b*, which is defined between the *extended* radial line and the tangent to the curve.

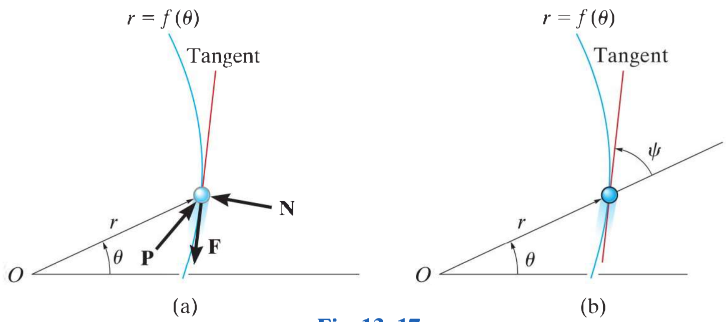
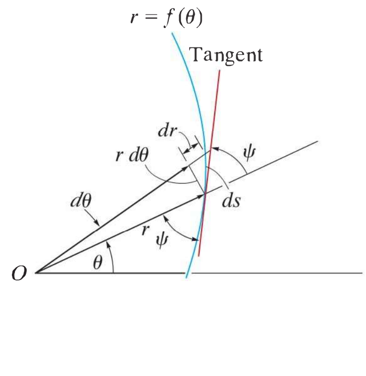


Fig. 13–17

This angle can be obtained by noting that when the particle is displaced a distance ds along the path, Fig. 13–17c, the component of displacement in the radial direction is dr and the component of displacement in the transverse direction is $r\,d\theta$. Since these two components are mutually perpendicular, the angle ψ can be determined from $\tan\psi = r\,d\theta/dr$, or

$$\tan\psi = \frac{r}{dr/d\theta} \tag{13–10}$$

If ψ is calculated as a positive quantity, it is measured from the *extended radial line* to the tangent in a counterclockwise sense or in the positive direction of θ. If it is negative, it is measured in the opposite direction to positive θ. For example, consider the cardioid $r = a(1 + \cos\theta)$, shown in Fig. 13–18. Because $dr/d\theta = -a\sin\theta$, then when $\theta = 30°$, $\tan\psi = a(1 + \cos 30°)/(-a\sin 30°) = -3.732$, or $\psi = -75°$, measured clockwise, opposite to $+\theta$ as shown in the figure.



(c)

Fig. 13–17

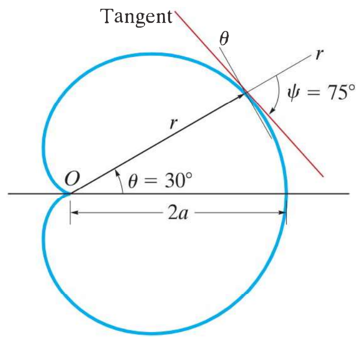


Fig. 13–18

Procedure for Analysis

Cylindrical or polar coordinates are a suitable choice for the analysis of a problem for which data regarding the angular motion of the radial line r are given, or in cases where the path can be conveniently expressed in terms of these coordinates. Once these coordinates have been established, the equations of motion can then be applied in order to relate the forces acting on the particle to its acceleration components. The method for doing this has been outlined in the procedure for analysis given in Sec. 13.4. The following is a summary of this procedure.

Free-Body Diagram.

- Establish the r, θ, z inertial coordinate system and draw the particle's free-body diagram.
- Assume that $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_z$ act in the *positive directions* of r, θ, z if they are unknown.
- Identify all the unknowns in the problem.

Equations of Motion.

- Apply the equations of motion, Eqs. 13–9.

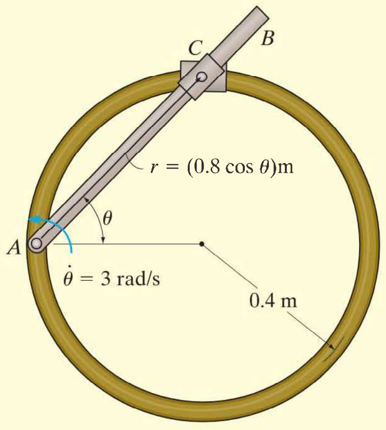
Kinematics.

- Use the methods of Sec. 12.8 to determine r and the time derivatives $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$, $\ddot{z}$, and then evaluate the acceleration components $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_z = \ddot{z}$.
- If any of the acceleration components is computed as a negative quantity, it indicates that it acts in its negative coordinate direction.
- When taking the time derivatives of $r = f(\theta)$, it is very important to use the chain rule of calculus, which is discussed at the end of Appendix C.

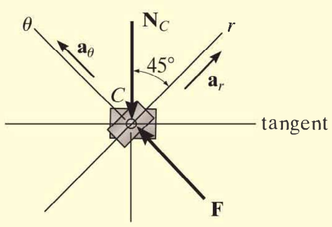
13

13

EXAMPLE 13.10



(a)



(b)

Fig. 13–19

The smooth 0.5-kg double-collar in Fig. 13–19*a* can freely slide on arm *AB* and the circular guide rod. If the arm rotates with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the force the arm exerts on the collar at the instant $\theta = 45°$. Motion is in the horizontal plane.

SOLUTION

Free-Body Diagram. The normal reaction $\mathbf{N}_C$ of the circular guide rod and the force $\mathbf{F}$ of arm AB act on the collar in the plane of motion, Fig. 13–19*b*. Note that $\mathbf{F}$ acts perpendicular to the axis of arm AB, that is, in the direction of the θ axis, while $\mathbf{N}_C$ acts perpendicular to the tangent of the circular path at $\theta = 45°$. The four unknowns are N_C, F, a_r, a_θ.

Equations of Motion.

$$+\nearrow\Sigma F_r = ma_r; \qquad -N_C \cos 45° = (0.5 \text{ kg})\, a_r \qquad (1)$$

$$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - N_C \sin 45° = (0.5 \text{ kg})\, a_\theta \qquad (2)$$

Kinematics. Using the chain rule (see Appendix C), the first and second time derivatives of r when $\theta = 45°, \dot{\theta} = 3$ rad/s, $\ddot{\theta} = 0$, are

$$r = 0.8 \cos \theta = 0.8 \cos 45° = 0.5657 \text{ m}$$

$$\dot{r} = -0.8 \sin \theta\, \dot{\theta} = -0.8 \sin 45°(3) = -1.6971 \text{ m/s}$$

$$\ddot{r} = -0.8\left[\sin \theta\, \ddot{\theta} + \cos \theta\, \dot{\theta}^2\right]$$

$$= -0.8[\sin 45°(0) + \cos 45°(3^2)] = -5.091 \text{ m/s}^2$$

We have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -5.091 \text{ m/s}^2 - (0.5657 \text{ m})(3 \text{ rad/s})^2 = -10.18 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.5657 \text{ m})(0) + 2(-1.6971 \text{ m/s})(3 \text{ rad/s})$$

$$= -10.18 \text{ m/s}^2$$

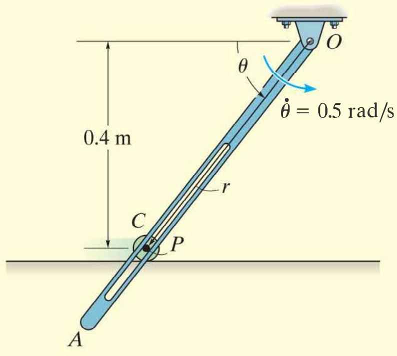
Substituting these results into Eqs. (1) and (2) and solving, we get

$$N_C = 7.20 \text{ N}$$

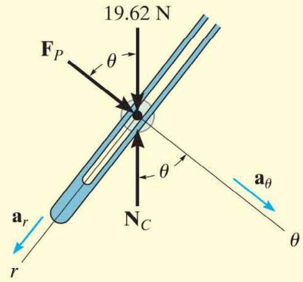
$$F = 0 \qquad \textit{Ans.}$$

EXAMPLE 13.11

The smooth 2-kg cylinder C in Fig. 13–20a has a pin P through its center which passes through the slot in arm OA. If the arm is forced to rotate in the *vertical plane* at a constant rate $\dot{\theta} = 0.5$ rad/s, determine the force that the arm exerts on the peg at the instant $\theta = 60°$.



(a)



(b)

Fig. 13–20

SOLUTION

Why is it a good idea to use polar coordinates to solve this problem?

Free-Body Diagram. The free-body diagram for the cylinder is shown in Fig. 13–20b. The force on the peg, $\mathbf{F}_P$, acts perpendicular to the slot in the arm. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the directions of *positive* r and θ, respectively. Identify the four unknowns.

Equations of Motion. Using the data in Fig. 13–20b, we have

$$+\swarrow \Sigma F_r = ma_r; \qquad 19.62 \sin\theta - N_C \sin\theta = 2a_r \qquad (1)$$

$$+\searrow \Sigma F_\theta = ma_\theta; \qquad 19.62 \cos\theta + F_P - N_C \cos\theta = 2a_\theta \qquad (2)$$

Kinematics. From Fig. 13–20a, r can be related to θ by the equation

$$r = \frac{0.4}{\sin\theta} = 0.4 \csc\theta$$

Since $d(\csc\theta) = -(\csc\theta \cot\theta)\, d\theta$ and $d(\cot\theta) = -(\csc^2\theta)\, d\theta$, then r and the necessary time derivatives become

$$\dot{\theta} = 0.5 \qquad r = 0.4 \csc\theta$$

$$\ddot{\theta} = 0 \qquad \dot{r} = -0.4(\csc\theta \cot\theta)\dot{\theta}$$

$$= -0.2 \csc\theta \cot\theta$$

$$\ddot{r} = -0.2(-\csc\theta\cot\theta)(\dot{\theta})\cot\theta - 0.2\csc\theta(-\csc^2\theta)\dot{\theta}$$

$$= 0.1 \csc\theta(\cot^2\theta + \csc^2\theta)$$

Evaluating these formulas at $\theta = 60°$, we get

$$\dot{\theta} = 0.5 \qquad r = 0.462$$

$$\ddot{\theta} = 0 \qquad \dot{r} = -0.133$$

$$\ddot{r} = 0.192$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.192 - 0.462(0.5)^2 = 0.0770$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.133)(0.5) = -0.133$$

Substituting these results into Eqs. 1 and 2 with $\theta = 60°$ and solving yields

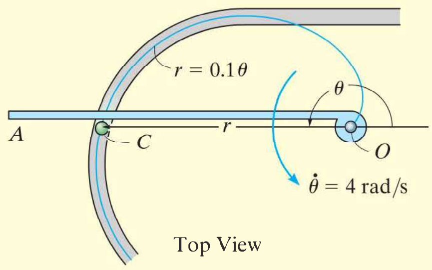
$$N_C = 19.5 \text{ N} \qquad F_P = -0.356 \text{ N} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{F}_P$ acts opposite to the direction shown in Fig. 13–20b.

13

EXAMPLE 13.12

A can C, having a mass of 0.5 kg, moves along a grooved horizontal slot shown in Fig. 13–21a. The slot is in the form of a spiral, which is defined by the equation $r = (0.1\theta)$ m, where θ is in radians. If the arm OA rotates with a constant rate $\dot{\theta} = 4$ rad/s in the horizontal plane, determine the force it exerts on the can at the instant $\theta = \pi$ rad. Neglect friction and the size of the can.



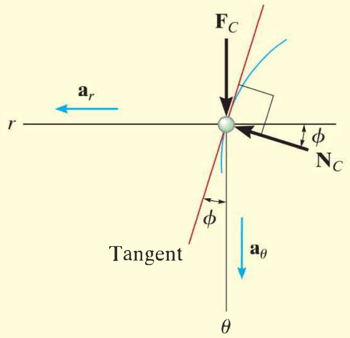
(a)

SOLUTION

Free-Body Diagram. The driving force $\mathbf{F}_C$ acts perpendicular to the arm OA, whereas the normal force of the wall of the slot on the can, $\mathbf{N}_C$, acts perpendicular to the tangent to the curve at $\theta = \pi$ rad, Fig. 13–21b. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the *positive directions* of r and θ, respectively. Since the path is specified, the angle ψ which the extended radial line r makes with the tangent, Fig. 13–21c, can be determined from Eq. 13–10. We have $r = 0.1\theta$, so that $dr/d\theta = 0.1$, and therefore

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.1\theta}{0.1} = \theta$$

When $\theta = \pi$, $\psi = \tan^{-1}\pi = 72.3°$, so that $\phi = 90° - \psi = 17.7°$, as shown in Fig. 13–21$c$. Identify the four unknowns in Fig. 13–21b.



(b)

Equations of Motion. Using $\phi = 17.7°$ and the data shown in Fig. 13–21b, we have

$$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad N_C \cos 17.7° = 0.5a_r \qquad (1)$$

$$+\downarrow \Sigma F_\theta = ma_\theta; \qquad F_C - N_C \sin 17.7° = 0.5a_\theta \qquad (2)$$

Kinematics. The time derivatives of r and θ are

$$\dot{\theta} = 4 \text{ rad/s} \qquad r = 0.1\theta$$

$$\ddot{\theta} = 0 \qquad \dot{r} = 0.1\dot{\theta} = 0.1(4) = 0.4 \text{ m/s}$$

$$\ddot{r} = 0.1\ddot{\theta} = 0$$

At the instant $\theta = \pi$ rad,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.1(\pi)(4)^2 = -5.03 \text{ m/s}^2$$

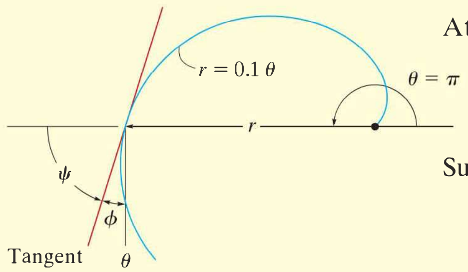
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.4)(4) = 3.20 \text{ m/s}^2$$

Substituting these results into Eqs. 1 and 2 and solving yields

$$N_C = -2.64 \text{ N}$$

$$F_C = 0.800 \text{ N} \qquad \textit{Ans.}$$

What does the negative sign for N_C indicate?

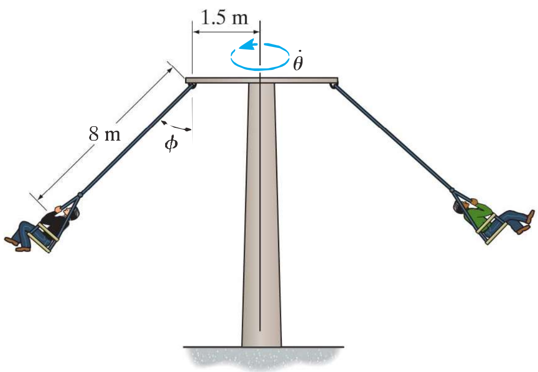


(c)

Fig. 13–21

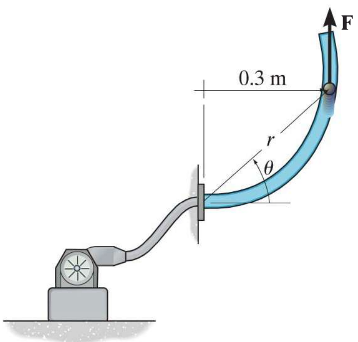
FUNDAMENTAL PROBLEMS

F13–13. Determine the constant angular velocity $\dot{\theta}$ of the vertical shaft of the amusement ride if $\phi = 45°$. Neglect the mass of the cables and the size of the passengers.



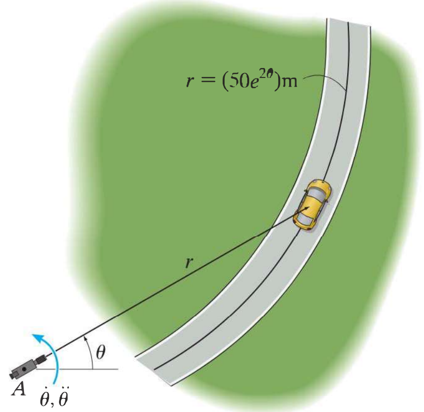
F13–13

F13–14. The 0.2-kg ball is blown through the smooth vertical circular tube whose shape is defined by $r = (0.6 \sin \theta)$ m, where θ is in radians. If $\theta = (\pi t^2)$ rad, where t is in seconds, determine the magnitude of force **F** exerted by the blower on the ball when $t = 0.5$ s.



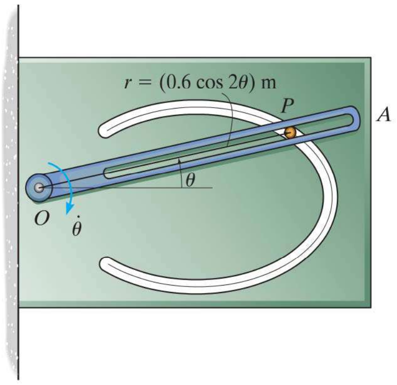
F13–14

F13–15. The 2-Mg car is traveling along the curved road described by $r = (50e^{2\theta})$ m, where θ is in radians. If a camera is located at A and it rotates with an angular velocity of $\dot{\theta} = 0.05$ rad/s and an angular acceleration of $\ddot{\theta} = 0.01$ rad/s^2 at the instant $\theta = \frac{\pi}{6}$ rad, determine the resultant friction force developed between the tires and the road at this instant.



F13–15

F13–16. The 0.2-kg pin P is constrained to move in the smooth curved slot, which is defined by the lemniscate $r = (0.6 \cos 2\theta)$ m. Its motion is controlled by the rotation of the slotted arm OA, which has a constant clockwise angular velocity of $\dot{\theta} = -3$ rad/s. Determine the force arm OA exerts on the pin P when $\theta = 0°$. Motion is in the vertical plane.



F13–16

PROBLEMS

***13–84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the resultant force acting on the particle when $t = 2$ s.

•13–85. Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

13–86. A 2-kg particle travels along a horizontal smooth path defined by

$$r = \left(\frac{1}{4}t^3 + 2\right)\text{m}, \ \theta = \left(\frac{t^2}{4}\right)\text{rad},$$

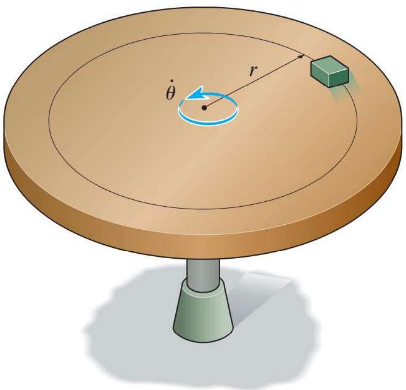
where t is in seconds. Determine the radial and transverse components of force exerted on the particle when $t = 2$ s.

13–87. A 2-kg particle travels along a path defined by

$$r = (3 + 2t^2)\,\text{m}, \ \theta = \left(\frac{1}{3}t^3 + 2\right)\text{rad}$$

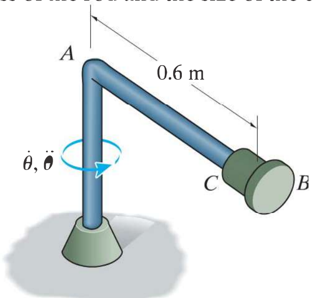
and $z = (5 - 2t^2)$ m, where t is in seconds. Determine the r, θ, z components of force that the path exerts on the particle at the instant $t = 1$ s.

***13–88.** If the coefficient of static friction between the block of mass m and the turntable is μ_s, determine the maximum constant angular velocity of the platform without causing the block to slip.



Prob. 13–88

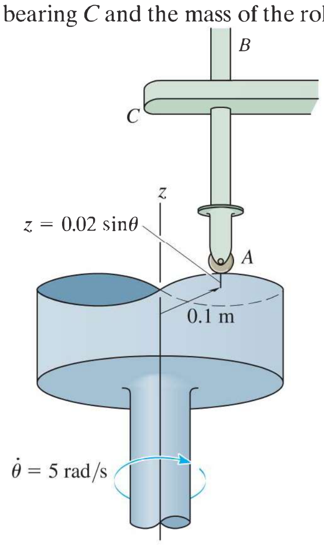
•13–89. The 0.5-kg collar C can slide freely along the smooth rod AB. At a given instant, rod AB is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and has an angular acceleration of $\ddot{\theta} = 2$ rad/s^2. Determine the normal force of rod AB and the radial reaction of the end plate B on the collar at this instant. Neglect the mass of the rod and the size of the collar.



Prob. 13–89

13–90. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin \theta)$ m. If the cam is rotating with a constant angular velocity of 5 rad/s, determine the force on the roller A when $\theta = 90°$. Neglect friction at the bearing C and the mass of the roller.

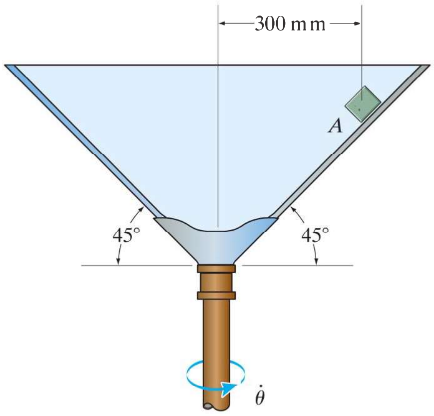
13–91. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin \theta)$ m. If the cam is rotating at a constant angular velocity of 5 rad/s, determine the maximum and minimum force the cam exerts on the roller at A. Neglect friction at the bearing C and the mass of the roller.



Probs. 13–90/91

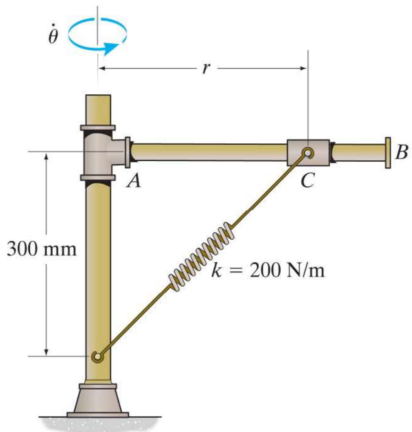
***13–92.** If the coefficient of static friction between the conical surface and the block of mass m is $\mu_s = 0.2$, determine the minimum constant angular velocity $\dot{\theta}$ so that the block does not slide downwards.

•13–93. If the coefficient of static friction between the conical surface and the block is $\mu_s = 0.2$, determine the maximum constant angular velocity $\dot{\theta}$ without causing the block to slide upwards.



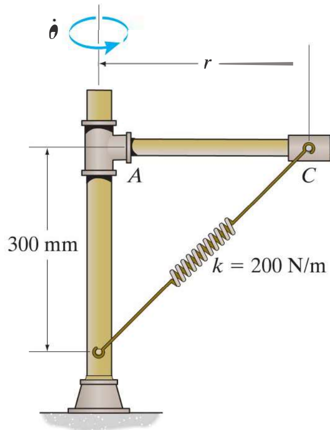
Probs. 13–92/93

13–94. If the position of the 3-kg collar C on the smooth rod AB is held at $r = 720$mm, determine the constant angular velocity $\dot{\theta}$ at which the mechanism is rotating about the vertical axis. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



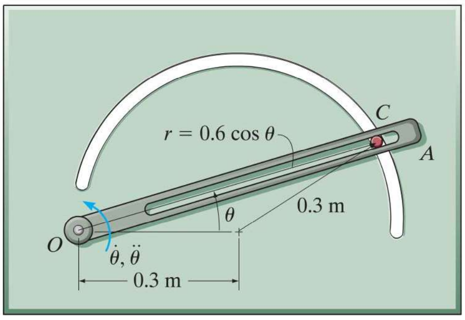
Prob. 13–94

13–95. The mechanism is rotating about the vertical axis with a constant angular velocity of $\dot{\theta} = 6$ rad/s. If rod AB is smooth, determine the constant position r of the 3-kg collar C. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



Prob. 13–95

***13–96.** Due to the constraint, the 0.5-kg cylinder C travels along the path described by $r = (0.6 \cos \theta)$ m. If arm OA rotates counterclockwise with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force exerted by the arm on the cylinder at this instant. The cylinder is in contact with only one edge of the smooth slot, and the motion occurs in the horizontal plane.

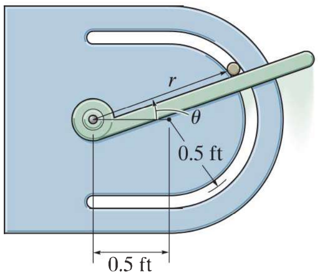


Prob. 13–96

13

•**13–97.** The 0.75-lb smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*

13–98. Solve Prob. 13–97 if motion occurs in the *vertical plane.*

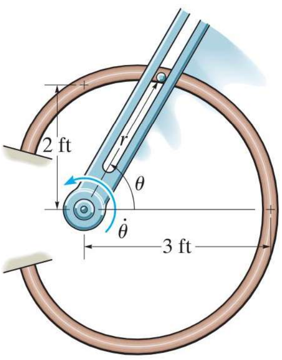


Probs. 13–97/98

13–99. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.

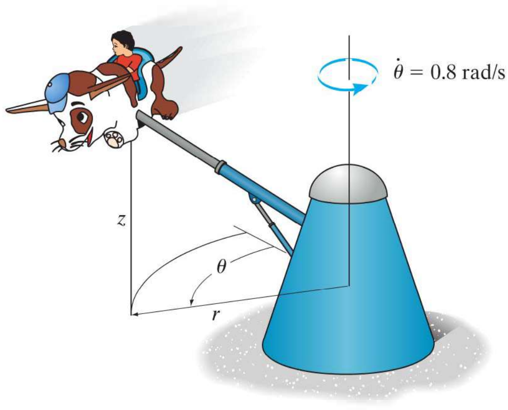
***13–100.** Solve Prob. 13–99 at the instant $\theta = 60°$.

•**13–101.** The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



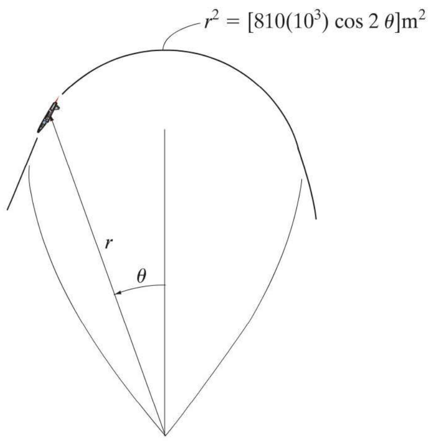
Probs. 13–99/100/101

13–102. The amusement park ride rotates with a constant angular velocity of $\dot{\theta} = 0.8$ rad/s. If the path of the ride is defined by $r = (3\sin\theta + 5)$ m and $z = (3\cos\theta)$ m, determine the r, θ, and z components of force exerted by the seat on the 20-kg boy when $\theta = 120°$.



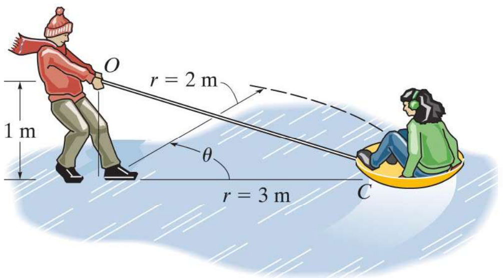
Prob. 13–102

13–103. The airplane executes the vertical loop defined by $r^2 = [810(10^3)\cos 2\theta]$ m^2. If the pilot maintains a constant speed $v = 120$ m/s along the path, determine the normal force the seat exerts on him at the instant $\theta = 0°$. The pilot has a mass of 75 kg.



Prob. 13–103

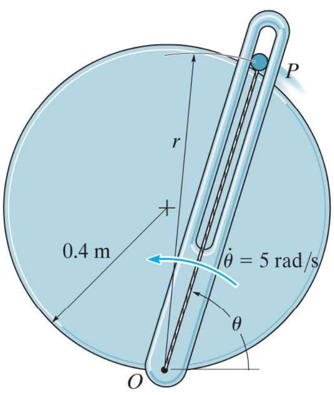
***13–104.** A boy standing firmly spins the girl sitting on a circular "dish" or sled in a circular path of radius $r_0 = 3$ m such that her angular velocity is $\dot{\theta}_0 = 0.1$ rad/s. If the attached cable OC is drawn inward such that the radial coordinate r changes with a constant speed of $\dot{r} = -0.5$ m/s, determine the tension it exerts on the sled at the instant $r = 2$ m. The sled and girl have a total mass of 50 kg. Neglect the size of the girl and sled and the effects of friction between the sled and ice. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\,d/dt(r^2\dot{\theta}) = 0$. When integrated, $r^2\dot{\theta} = C$, where the constant C is determined from the problem data.



Prob. 13–104

13–105. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin\theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.

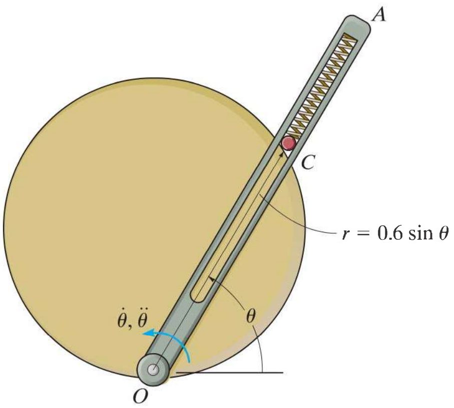
13–106. Solve Prob. 13–105 if $\ddot{\theta} = 2\text{ rad/s}^2$ when $\dot{\theta} = 5$ rad/s and $\theta = 60°$.



Probs. 13–105/106

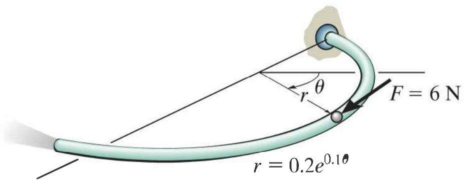
13–107. The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin\theta)$ m. If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the horizontal plane.

***13–108.** The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin\theta)$ m. If arm OA is rotating counterclockwise with an angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the vertical plane.



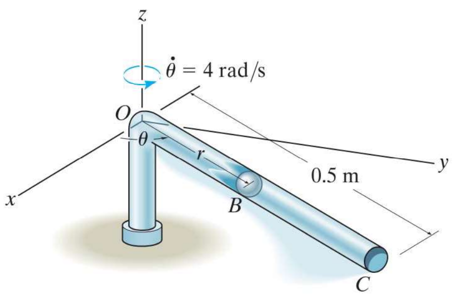
Probs. 13–107/108

•13–109. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the horizontal plane and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to air pressure is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. Also, what is the angle ψ from the extended radial coordinate r to the line of action of the 6-N force?



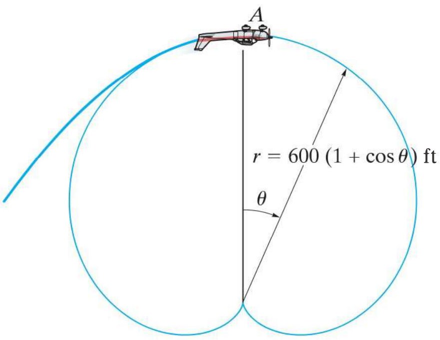
Prob. 13–109

13–110. The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity of $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.



Prob. 13–110

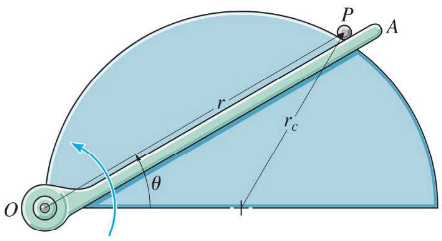
13–111. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft. If his speed at A $(\theta = 0°)$ is a constant $v_P = 80$ ft/s, determine the vertical force the seat belt must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb.



Prob. 13–111

***13–112.** The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.

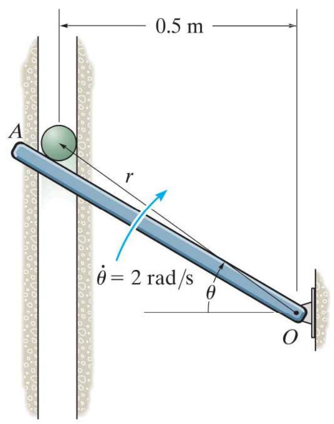
•**13–113.** The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



Probs. 13–112/113

13–114. The ball has a mass of 1 kg and is confined to move along the smooth vertical slot due to the rotation of the smooth arm OA. Determine the force of the rod on the ball and the normal force of the slot on the ball when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 3$ rad/s. Assume the ball contacts only one side of the slot at any instant.

13–115. Solve Prob. 13–114 if the arm has an angular acceleration of $\ddot{\theta} = 2$ rad/s^2 when $\dot{\theta} = 3$ rad/s at $\theta = 30°$.



Probs. 13–114/115