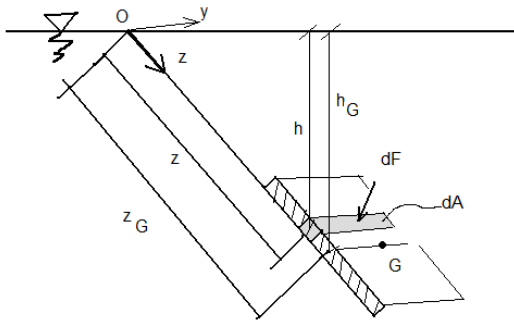


FLUID MECHANICS

THE FORCES APPLIED TO SUBMERGED PLANE SURFACES



Pressure Force: A force will affect the surface that in a static fluid. This force will vary depending on the distribution of pressure. This force is called Pressure Force (Hydrostatic Force) and the point of application is called Pressure Center.

Let's find the formula;

$$P = F/A \Rightarrow F = P A$$

$$dF = \rho g h dA$$

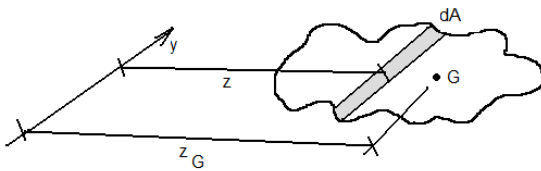
{P = \rho g h} {h = z \sin\theta} After writing places then both sides can be integrated

$$\int dF = \int \rho g z \sin\theta dA$$

$$F = \int \rho g z \sin\theta dA$$

$$F = \rho g \sin\theta \int z dA$$

There is no connection between the variables z and dA. According to let's find another way to save from integrals



Referring to the figure on the left side, write the balance of torque based on the y-axis.

$$Z_G A = \int z dA$$

This expression is typed instead of the above formula

$$F = \rho g \sin\theta Z_G A$$

$$F = \rho g \sin\theta Z_G A$$

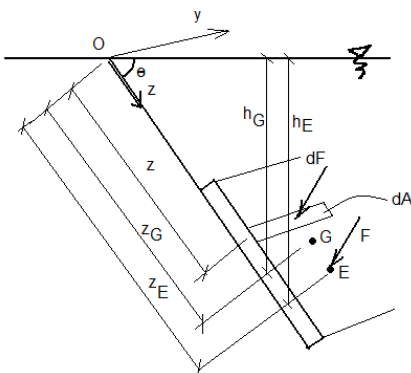
$$\{h_G = \sin\theta Z_G\}$$

$$\boxed{F = \rho g h_G A}$$

Accordingly, the pressure force is equal to multiplication of the pressure in the center of gravity and surface area.

Pressure Center: Although, pressure force has been computed by using the gravity center, this center is a point below the center of gravity, due to the increasing pressure as the depth increases.

To find the center of pressure, the y-axis torque of distributed pressure load on the plane should be equal to the torque based on the same axis of the center of pressure in E point.



{ Pressure Force Torque } = { Distributed Pressure Force Torque }

$$F z_E = \int dF z$$

$$F z_E = \int [(\rho g z \sin\theta dA) z]$$

$$F z_E = \int \rho g z^2 \sin\theta dA$$

$$F z_E = \rho g \sin\theta \int z^2 dA$$

{ moment of inertia formula $I_y = \int z^2 dA$ }

$$F z_E = \rho g \sin\theta I_y$$

$$(\rho g h_G A) z_E = \rho g \sin\theta I_y$$

$$(\rho g h_G A) z_E = \rho g \sin\theta I_y$$

{ $h_G = z_G \sin\theta$ }

$$\rho g z_G \sin\theta A z_E = \rho g \sin\theta I_y$$

$$z_E = \frac{I_y}{z_G A}$$

{ According to any axis $I_y = I_G + z_G^2 A$ }

$$e = z_E - z_G = (I_G + z_G^2 A) / (z_G A) - z_G$$

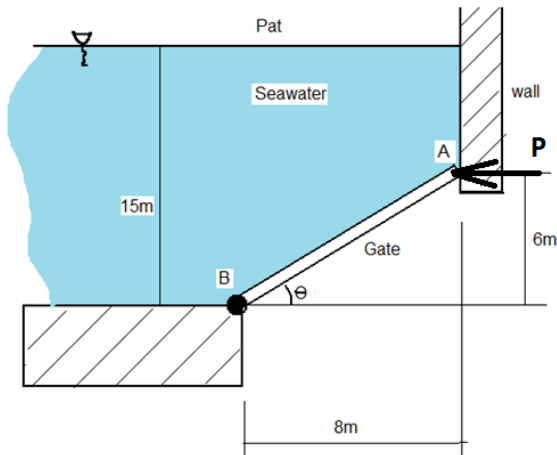
$$e = \frac{I_G}{z_G A}$$

$$e = \frac{I_G}{h_G A}$$

Attention: Z_G

Example: The gate in Fig. is 5 m wide. It is hinged at point B, and rests against a smooth wall at point A. Compute

- (a) the Force on the gate due to seawater pressure
- (b) the horizontal force P exerted by the wall at point A and
- (c) the reaction forces at the hinge B.



- a) By geometry the gate is 10 m long from A to B, and its centroid is halfway between, or elevation 3m above point B. The depth h_G is thus $15-3=12$ m. The gate area is $5\text{ m} \times 10\text{ m} = 50\text{ m}^2$. Neglect P_a as acting on both sides of the gate. The hydrostatic force on the gate is

$$F = P_G A = \rho g h_G A = 1020\text{ kg/m}^3 \cdot 9.81\text{ m/s}^2 \cdot 12\text{ m} \cdot 50\text{ m}^2$$

$$F = 6,003,720\text{ N} = 6003\text{ kN.}$$

- b) First we must find the center of pressure of F. A free body diagram of the gate is shown Figure. The gate is rectangle hence,

$$I_G = (b L^3) / 12 = 5\text{ m} \cdot 10^3\text{ m}^3 / 12 = 417\text{ m}^4$$

The distance e from G to E is given Equation before.

$$e = \frac{I_G}{z_G A} = 417 / [(h_G / \sin\theta) 50] = 417 / [(12\text{ m} / (3/5)) 50] = 0.417\text{ m}$$

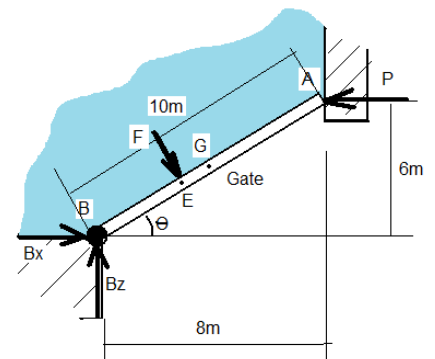
The distance from point B to Force F is thus $10 - e = 9.583\text{ m}$.

Summing moments counter clock wise about B gives;

$$P L \sin\theta - F (5 - e) = 0$$

$$P \cdot 6\text{ m} - 6003720\text{ N} \cdot 4.583\text{ m}$$

$$P = 4585\text{ kN.}$$



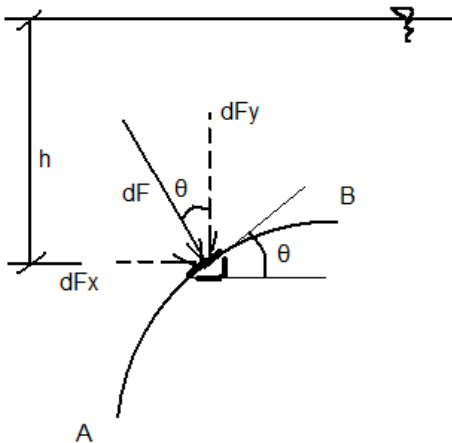
- c) With F and P known. The reactions Bx and Bz are found by summing forces on the Gate.

$$\Sigma F_x = 0 \Rightarrow B_x + F \sin\theta - P = 0 \Rightarrow B_x = -6003720\text{ N} (3/5) + 4585000\text{ N} \Rightarrow B_x = 982768\text{ N (towards right)}$$

$$\Sigma F_z = 0 \Rightarrow B_z - F \cos\theta = 0 \Rightarrow B_z = 6003720\text{ N} (4/5) = 4802976\text{ N (towards up)}$$

THE FORCES APPLIED TO CURVED SUBMERGED SURFACES

The calculation of the pressure force on a curved surface, the horizontal and vertical components are considered separately.



Elemental force dF to the elemental area dA is;

$$dF = P \, dA$$

It can be written as y component;

$$dF_y = P \, dA \, \cos\theta$$

$$dF_y = (\rho g h) \, dA \, \cos\theta$$

Here, $dA \cos\theta$ expression is the horizontal projection of the dA area.

Accordingly $(h \, dA \, \cos\theta)$ expression is the volume of column for this area.

Then we found the weight of the column by multiplying with (ρg) .

In this way, Is taken the sum of all the infinitesimal areas on AB area (in other words, by getting integral), it is found the weight of fluid on the AB area.

Briefly, the **y component of the force action on a curved surface, equal to the weight of fluid on this surface.**

Let now found x component of the dF force. If we write the same way;

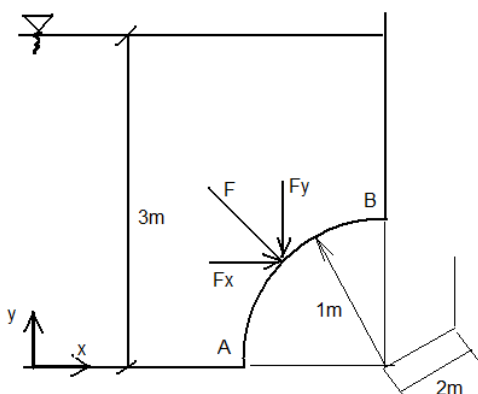
$$dF_x = P \, dA \, \sin\theta$$

Here, $dA \sin\theta$ expression is the vertical projection of the dA area.

Thus, the horizontal force of the dA area is equal to the force of the pressure acting to perpendicular projection of this field.

In this way, the AB surface is scanned complete, the horizontal force of acting on AB is found.

Briefly, the **horizontal force to the AB is equal to the hydrostatic force acting to vertical projection to this AB surface.**



It is generally not possible, to represent the forces acting on a curved surface by the force action on a particular point on a curved surface. In other words, It is not searched the center of pressure on curved surfaces. So there is no Pressure Center in curved surfaces.

Example: Find the vertical force and horizontal force on the quarter circular door AB, shown in Figure. Water acting on one side and air on the other. The door is 2 m in width .

Solution: For the horizontal force F_x , we project the surface of the curved door onto a plane parallel to the xy plane. This projected area is a 1 by 2 m rectangle shown on edge as OB in Figure. We may now use formulation for a plane surface to determine F_x . The atmospheric pressure on the free surface clearly develops a horizontal force on the left side of the door AB that is canceled completely by the horizontal force from the

atmospheric on the right side of the door. We need worry only about the gravitational effect on the water.

We then have

$$F_x = P A = (\rho g h) A = 1000\text{kg/m}^3 \cdot 9.81\text{m/s}^2 \cdot 2.5\text{m} \cdot 1\text{m} \cdot 2\text{m} = 49,030 \text{ N} = 49 \text{ kN}.$$

As for the vertical component, we need only consider the weight of the column of water directly above the door AB, We thus have (see figure);

$$F_y = \rho g (V) = 1000\text{kg/m}^3 \cdot 9.81\text{m/s}^2 [(3\text{m} \cdot 1\text{m} \cdot 2\text{m}) - (\pi r^2 / 4 \text{m}^2 \cdot 2\text{m})] = 43,400 \text{ N} = 43.4 \text{ kN}$$

The resultant force is then

$$F = \sqrt{(43.4^2 + 49^2)} = 65.46 \text{ kN}$$

Because the pressure forces are at all times perpendicular to the circular AB, it should be clear the simplest resultant has a line of action passing through point O.

SOLVED QUESTIONS

Example

Example

Consider the hypothetical Figure 4.24. The last layer is made of water with density of $1000[\text{kg/m}^3]$. The densities are $\rho_1 = 500[\text{kg/m}^3]$, $\rho_2 = 800[\text{kg/m}^3]$, $\rho_3 = 850[\text{kg/m}^3]$, and $\rho_4 = 1000[\text{kg/m}^3]$. Calculate the forces at points a_1 and b_1 . Assume that the layers are stables without any movement between the liquids. Also neglect all mass transfer phenomena that may occur. The heights are: $h_1 = 1[\text{m}]$, $h_2 = 2[\text{m}]$, $h_3 = 3[\text{m}]$, and $h_4 = 4[\text{m}]$. The forces distances are $a_1 = 1.5[\text{m}]$, $a_2 = 1.75[\text{m}]$, and $b_1 = 4.5[\text{m}]$. The angle of inclination is $\beta = 45^\circ$.

SOLUTION

Since there are only two unknowns, only two equations are needed, which are (4.133) and (4.130). The solution method of this example is applied for cases with less layers (for example by setting the specific height difference to be zero). Equation (4.133) can be used by modifying it, as it can be noticed that instead of using the regular atmospheric pressure the new "atmospheric" pressure can be used as

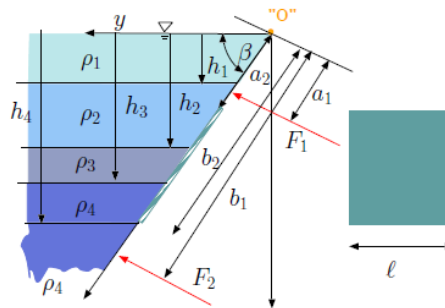


Fig. -4.24. The effects of multi layers density on static forces.

$$P_{atmos}' = P_{atmos} + \rho_1 g h_1$$

The distance for the center for each area is at the middle of each of the “small” rectangular. The geometries of each areas are

$$\begin{aligned} x_{c1} &= \frac{a_2 + \frac{h_2}{\sin \beta}}{2} & A_1 &= \ell \left(\frac{h_2}{\sin \beta} - a_2 \right) & I_{x'x'_1} &= \frac{\ell \left(\frac{h_2}{\sin \beta} - a_2 \right)^3}{36} + (x_{c1})^2 A_1 \\ x_{c2} &= \frac{h_2 + h_3}{2 \sin \beta} & A_2 &= \frac{\ell}{\sin \beta} (h_3 - h_2) & I_{x'x'_2} &= \frac{\ell (h_3 - h_2)^3}{36 \sin \beta} + (x_{c2})^2 A_2 \\ x_{c3} &= \frac{h_3 + h_4}{2 \sin \beta} & A_3 &= \frac{\ell}{\sin \beta} (h_4 - h_3) & I_{x'x'_3} &= \frac{\ell (h_4 - h_3)^3}{36 \sin \beta} + (x_{c3})^2 A_3 \end{aligned}$$

After inserting the values, the following equations are obtained

Thus, the first equation is

$$F_1 + F_2 = P_{atmos}' \overbrace{\ell(b_2 - a_2)}^{A_{total}} + g \sin \beta \sum_{i=1}^3 \rho_{i+1} x_{ci} A_i$$

The second equation is (4.133) to be written for the moment around the point “O” as

$$F_1 a_1 + F_2 b_1 = P_{atmos}' \overbrace{\frac{(b_2 + a_2)}{2} \ell(b_2 - a_2)}^{x_c A_{total}} + g \sin \beta \sum_{i=1}^3 \rho_{i+1} I_{x'x'_i}$$

The solution for the above equation is

$$\begin{aligned} F_1 &= \frac{2 b_1 g \sin \beta \sum_{i=1}^3 \rho_{i+1} x_{ci} A_i - 2 g \sin \beta \sum_{i=1}^3 \rho_{i+1} I_{x'x'_i}}{2 b_1 - 2 a_1} \\ &= \frac{(b_2^2 - 2 b_1 b_2 + 2 a_2 b_1 - a_2^2) \ell P_{atmos}}{2 b_1 - 2 a_1} \\ F_2 &= \frac{2 g \sin \beta \sum_{i=1}^3 \rho_{i+1} I_{x'x'_i} - 2 a_1 g \sin \beta \sum_{i=1}^3 \rho_{i+1} x_{ci} A_i}{2 b_1 - 2 a_1} + \\ &= \frac{(b_2^2 + 2 a_1 b_2 + a_2^2 - 2 a_1 a_2) \ell P_{atmos}}{2 b_1 - 2 a_1} \end{aligned}$$

The solution provided isn't in the complete long form since it will makes things messy. It is simpler to compute the terms separately. A mini source code for the calculations is provided in the the text source. The intermediate results in SI units ([m], [m²], [m⁴]) are:

$$\begin{aligned} x_{c1} &= 2.2892 & x_{c2} &= 3.5355 & x_{c3} &= 4.9497 \\ A_1 &= 2.696 & A_2 &= 3.535 & A_3 &= 3.535 \\ I_{x'x'_1} &= 14.215 & I_{x'x'_2} &= 44.292 & I_{x'x'_3} &= 86.718 \end{aligned}$$

The final answer is

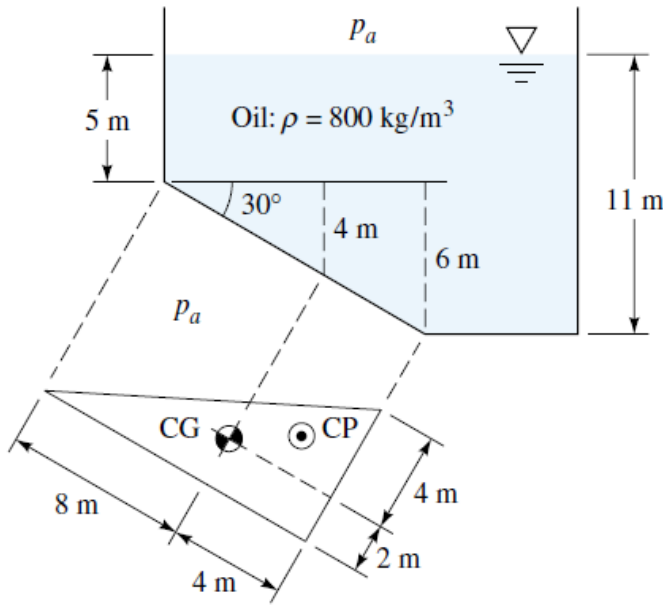
$$F_1 = 304809.79[N]$$

and

$$F_2 = 958923.92[N]$$

Example

A tank of oil has a right-triangular panel near the bottom, as in Fig. E2.6. Omitting p_a , find the (a) hydrostatic force and (b) CP on the panel.



E2.6

Part (a) The triangle has properties given in Fig. 2.13c. The centroid is one-third up (4 m) and one-third over (2 m) from the lower left corner, as shown. The area is

$$\frac{1}{2}(6 \text{ m})(12 \text{ m}) = 36 \text{ m}^2$$

The moments of inertia are

$$I_{xx} = \frac{bL^3}{36} = \frac{(6 \text{ m})(12 \text{ m})^3}{36} = 288 \text{ m}^4$$

and
$$I_{xy} = \frac{b(b - 2s)L^2}{72} = \frac{(6 \text{ m})[6 \text{ m} - 2(6 \text{ m})](12 \text{ m})^2}{72} = -72 \text{ m}^4$$

The depth to the centroid is $h_{CG} = 5 + 4 = 9 \text{ m}$; thus the hydrostatic force from Eq. (2.44) is

$$\begin{aligned} F &= \rho gh_{CG}A = (800 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(9 \text{ m})(36 \text{ m}^2) \\ &= 2.54 \times 10^6 \text{ (kg} \cdot \text{m)/s}^2 = 2.54 \times 10^6 \text{ N} = 2.54 \text{ MN} \end{aligned} \quad \text{Ans. (a)}$$

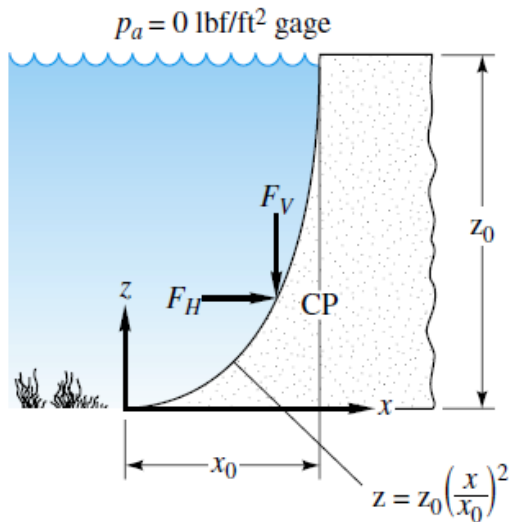
Part (b) The CP position is given by Eqs. (2.44):

$$\begin{aligned} y_{CP} &= -\frac{I_{xx} \sin \theta}{h_{CG}A} = -\frac{(288 \text{ m}^4)(\sin 30^\circ)}{(9 \text{ m})(36 \text{ m}^2)} = -0.444 \text{ m} \\ x_{CP} &= -\frac{I_{xy} \sin \theta}{h_{CG}A} = -\frac{(-72 \text{ m}^4)(\sin 30^\circ)}{(9 \text{ m})(36 \text{ m}^2)} = +0.111 \text{ m} \end{aligned} \quad \text{Ans. (b)}$$

The resultant force $F = 2.54 \text{ MN}$ acts through this point, which is down and to the right of the centroid, as shown in Fig. E2.6.

Example

A dam has a parabolic shape $z/z_0 = (x/x_0)^2$ as shown in Fig. E2.7a, with $x_0 = 10$ ft and $z_0 = 24$ ft. The fluid is water, $\gamma = 62.4$ lbf/ft³, and atmospheric pressure may be omitted. Compute the



E2.7a

forces F_H and F_V on the dam and the position CP where they act. The width of the dam is 50 ft.

Solution

The vertical projection of this curved surface is a rectangle 24 ft high and 50 ft wide, with its centroid halfway down, or $h_{CG} = 12$ ft. The force F_H is thus

$$F_H = \gamma h_{CG} A_{proj} = (62.4 \text{ lbf/ft}^3)(12 \text{ ft})(24 \text{ ft})(50 \text{ ft}) = 899,000 \text{ lbf} = 899 \times 10^3 \text{ lbf} \quad \text{Ans.}$$

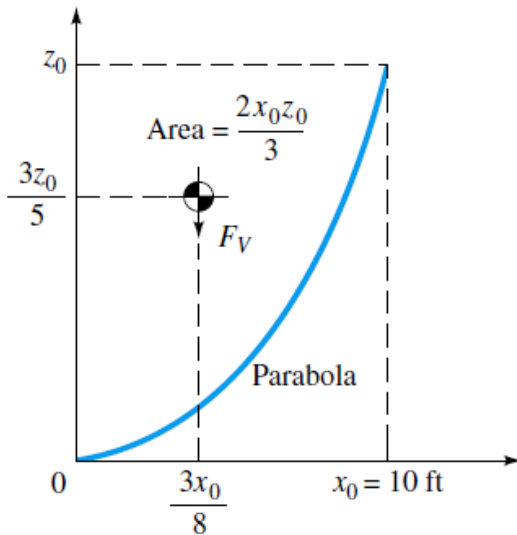
The line of action of F_H is below the centroid by an amount

$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A_{proj}} = -\frac{\frac{1}{12}(50 \text{ ft})(24 \text{ ft})^3(\sin 90^\circ)}{(12 \text{ ft})(24 \text{ ft})(50 \text{ ft})} = -4 \text{ ft}$$

Thus F_H is $12 + 4 = 16$ ft, or two-thirds, down from the free surface or 8 ft from the bottom, as might have been evident by inspection of the triangular pressure distribution.

The vertical component F_V equals the weight of the parabolic portion of fluid above the curved surface. The geometric properties of a parabola are shown in Fig. E2.7b. The weight of this amount of water is

$$F_V = \gamma \left(\frac{2}{3}x_0 z_0 b\right) = (62.4 \text{ lbf/ft}^3)\left(\frac{2}{3}\right)(10 \text{ ft})(24 \text{ ft})(50 \text{ ft}) = 499,000 \text{ lbf} = 499 \times 10^3 \text{ lbf} \quad \text{Ans.}$$



E2.7b

This acts downward on the surface at a distance $3x_0/8 = 3.75$ ft over from the origin of coordinates. Note that the vertical distance $3z_0/5$ in Fig. E2.7b is irrelevant.

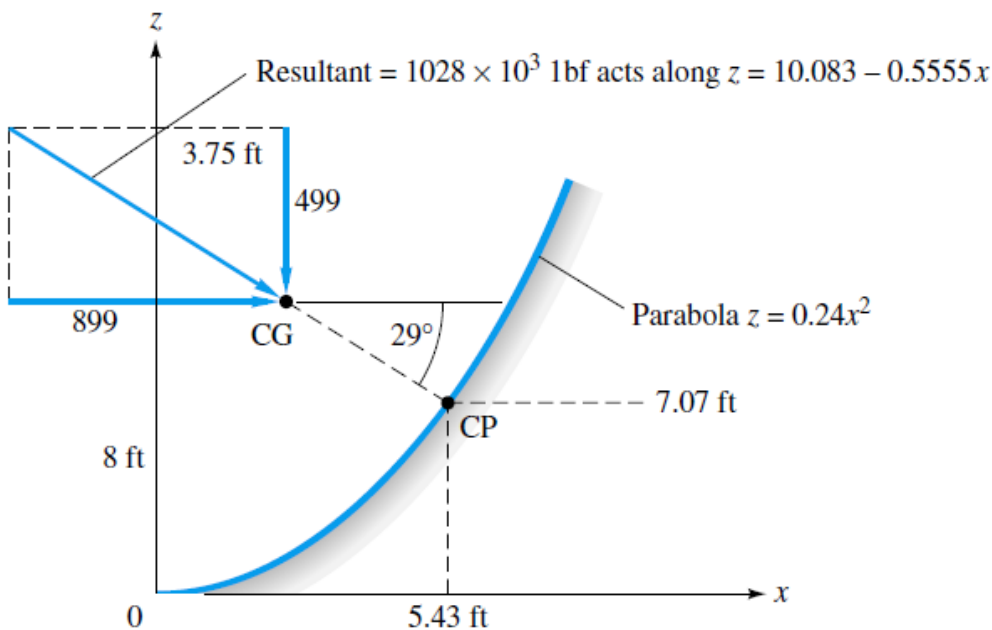
The total resultant force acting on the dam is

$$F = (F_H^2 + F_V^2)^{1/2} = [(499)^2 + (899)^2]^{1/2} = 1028 \times 10^3 \text{ lbf}$$

As seen in Fig. E2.7c, this force acts down and to the right at an angle of $29^\circ = \tan^{-1} \frac{499}{899}$. The force F passes through the point $(x, z) = (3.75 \text{ ft}, 8 \text{ ft})$. If we move down along the 29° line until we strike the dam, we find an equivalent center of pressure on the dam at

$$x_{CP} = 5.43 \text{ ft} \quad z_{CP} = 7.07 \text{ ft} \quad \text{Ans.}$$

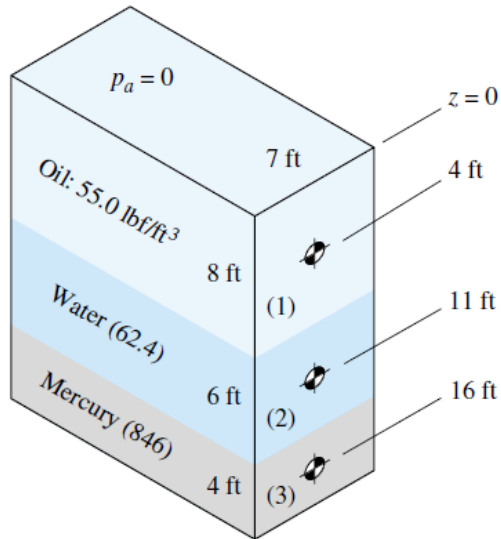
This definition of CP is rather artificial, but this is an unavoidable complication of dealing with a curved surface.



E2.7c

Example

A tank 20 ft deep and 7 ft wide is layered with 8 ft of oil, 6 ft of water, and 4 ft of mercury. Compute (a) the total hydrostatic force and (b) the resultant center of pressure of the fluid on the right-hand side of the tank.



E2.8

Part (a) Divide the end panel into three parts as sketched in Fig. E2.8, and find the hydrostatic pressure at the centroid of each part, using the relation (2.38) in steps as in Fig. E2.8:

$$P_{CG_1} = (55.0 \text{ lbf/ft}^3)(4 \text{ ft}) = 220 \text{ lbf/ft}^2$$

$$P_{CG_2} = (55.0)(8) + 62.4(3) = 627 \text{ lbf/ft}^2$$

$$P_{CG_3} = (55.0)(8) + 62.4(6) + 846(2) = 2506 \text{ lbf/ft}^2$$

These pressures are then multiplied by the respective panel areas to find the force on each portion:

$$F_1 = p_{CG_1}A_1 = (220 \text{ lbf/ft}^2)(8 \text{ ft})(7 \text{ ft}) = 12,300 \text{ lbf}$$

$$F_2 = p_{CG_2}A_2 = 627(6)(7) = 26,300 \text{ lbf}$$

$$F_3 = p_{CG_3}A_3 = 2506(4)(7) = \underline{70,200 \text{ lbf}}$$

$$F = \sum F_i = 108,800 \text{ lbf} \quad \text{Ans. (a)}$$

Part (b) Equations (2.47) can be used to locate the CP of each force F_i , noting that $\theta = 90^\circ$ and $\sin \theta = 1$ for all parts. The moments of inertia are $I_{xx_1} = (7 \text{ ft})(8 \text{ ft})^3/12 = 298.7 \text{ ft}^4$, $I_{xx_2} = 7(6)^3/12 = 126.0 \text{ ft}^4$, and $I_{xx_3} = 7(4)^3/12 = 37.3 \text{ ft}^4$. The centers of pressure are thus at

$$y_{CP_1} = -\frac{\rho_1 g I_{xx_1}}{F_1} = -\frac{(55.0 \text{ lbf/ft}^3)(298.7 \text{ ft}^4)}{12,300 \text{ lbf}} = -1.33 \text{ ft}$$

$$y_{CP_2} = -\frac{62.4(126.0)}{26,300} = -0.30 \text{ ft} \quad y_{CP_3} = -\frac{846(37.3)}{70,200} = -0.45 \text{ ft}$$

This locates $z_{CP_1} = -4 - 1.33 = -5.33 \text{ ft}$, $z_{CP_2} = -11 - 0.30 = -11.30 \text{ ft}$, and $z_{CP_3} = -16 - 0.45 = -16.45 \text{ ft}$. Summing moments about the surface then gives

$$\sum F_i z_{CP_i} = F z_{CP}$$

or $12,300(-5.33) + 26,300(-11.30) + 70,200(-16.45) = 108,800 z_{CP}$

or $z_{CP} = -\frac{1,518,000}{108,800} = -13.95 \text{ ft} \quad \text{Ans. (b)}$

The center of pressure of the total resultant force on the right side of the tank lies 13.95 ft below the surface.

Finding the Location of the Centroid

A second problem associated with the topic of curved surfaces is that of finding the location of the centroid of V_{eff} .

Recall:

Centroid = the location where the first moment of a point area, volume, or mass equals the first moment of the distributed area, volume, or mass, e.g.

$$x_{cg} V_1 = \int_V x dV$$

This principle can also be used to determine the location of the centroid of complex geometries.

For example:

If $V_{eff} = V_1 + V_2$

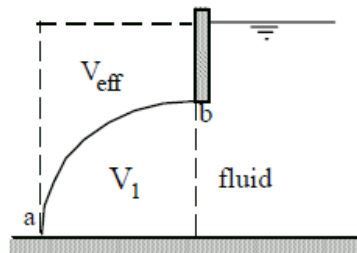
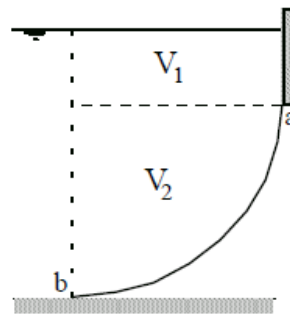
then

$x_{cg}V_{eff} = x_1V_1 + x_2V_2$

or

$V_T = V_1 + V_{eff}$

$x_TV_T = x_1V_1 + x_{cg}V_{eff}$

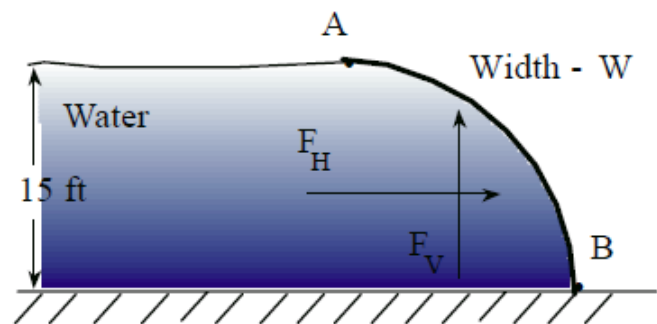


Note: In the figures shown above, each of the x values would be specified relative to a vertical axis through b since the cg of the quarter circle is most easily specified relative to this axis.

Example

Example:

Gate AB holds back 15 ft of water. Neglecting the weight of the gate, determine the magnitude (per unit width) and location of the hydrostatic forces on the gate and the resisting moment about B.

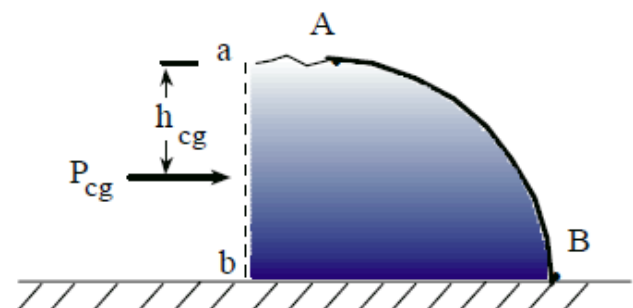


a. Horizontal component

$\gamma = \rho g = 62.4 \text{ lbf/ft}^3$

Rule: Project the curved surface into the vertical plane. Locate the centroid of the projected area. Find the pressure at the centroid of the vertical projection. $F = P_{cg} A_p$

Note: All calculations are done with the projected area. The curved surface is not used at all in the analysis.



The curved surface projects onto plane a - b and results in a **rectangle**, (not a quarter circle) 15 ft x W. For this rectangle:

$$h_{cg} = 7.5, \quad P_{cg} = \gamma h_{cg} = 62.4 \text{ lbf/ft}^3 * 7.5 \text{ ft} = 468 \text{ lbf/ft}^2$$

$$F_h = P_{cg} A = 468 \text{ lbf/ft}^2 * 15 \text{ ft} * W = \underline{7020 W \text{ lbf}} \quad \longrightarrow$$

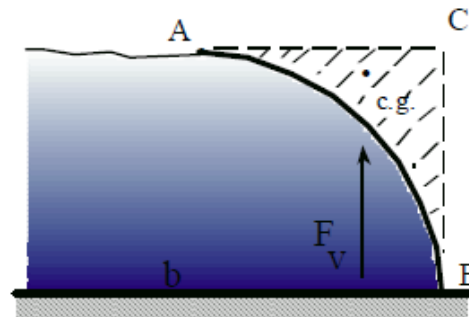
$$\text{Location: } I_{xx} = bh^3/12 = W * 15^3 /12 = 281.25 W \text{ ft}^4$$

$$y_{cp} = -\frac{I_{xx} \sin \theta}{h_{cg} A} = -\frac{281.25 W \text{ ft}^4 \sin 90^\circ}{7.5 \text{ ft} * 15 W \text{ ft}^2} = -2.5 \text{ ft}$$

The location is 2.5 ft below the c.g. or 10 ft below the surface, 5 ft above the bottom.

b. Vertical force:

Rule: F_v equals the weight of the effective column of fluid above the curved surface.



Q: What is the effective volume of fluid above the surface?

What volume of fluid would result in the actual pressure distribution on the curved surface?

$$Vol = A - B - C$$

$$V_{rec} = V_{qc} + V_{ABC}, \quad V_{ABC} = V_{rec} - V_{qc}$$

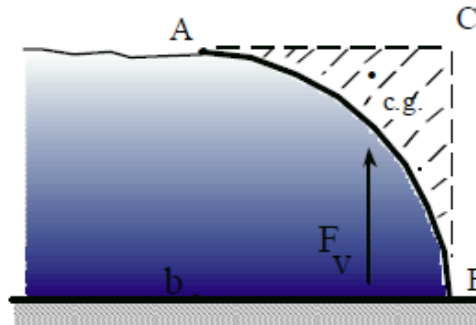
$$V_{ABC} = V_{eff} = 15^2 W - \pi 15^2/4 * W = 48.29 W \text{ ft}^3$$

$$F_v = \rho g V_{eff} = 62.4 \text{ lbf/ft}^3 * 48.29 \text{ ft}^3 = \underline{3013 \text{ lbf}} \quad \uparrow$$

Note: F_v is directed upward even though the effective volume is above the surface.

c. What is the location?

Rule: F_v will act through the centroid of the “effective volume causing the force.”



We need the centroid of volume A-B-C. How do we obtain this centroid?

Use the concept which is the basis of the centroid, the “first moment of an area.”

$$\text{Since: } A_{\text{rec}} = A_{\text{qc}} + A_{\text{ABC}} \quad M_{\text{rec}} = M_{\text{qc}} + M_{\text{ABC}} \quad M_{\text{ABC}} = M_{\text{rec}} - M_{\text{qc}}$$

Note: We are taking moments about the left side of the figure, ie., point b. **WHY?**

(The c.g. of the quarter circle is known to be $4R/3\pi$ w.r.t. b.)

$$x_{\text{cg}} A = x_{\text{rec}} A_{\text{rec}} - x_{\text{qc}} A_{\text{qc}}$$

$$x_{\text{cg}} \{15^2 - \pi \cdot 15^2/4\} = 7.5 \cdot 15^2 - \{4 \cdot 15/3/\pi\} \cdot \pi \cdot 15^2/4$$

$$x_{\text{cg}} = \mathbf{11.65 \text{ ft}} \quad \{ \text{distance to rt. of b to centroid} \}$$

Q: Do we need a y location? Why?

d. Calculate the moment about B needed for equilibrium.

$$\sum M_B = 0 \quad \text{clockwise positive.}$$

$$M_B + 5 F_h + (15 - x_v) F_v = 0$$

$$M_B + 5 \times 7020 W + (15 - 11.65) 3013 W = 0 \quad M_B + 5 \times 7020 W + (15 - 11.65) 3013 W = 0$$

$$P_a \neq \rho g y \quad G \neq g$$

$$M_B + 35,100 W + 10,093.6 W = 0$$

$$\underline{M_B = -45,194 W \text{ ft-lbf}} \quad \text{Why negative?}$$

The hydrostatic forces will tend to roll the surface clockwise relative to B, thus a resisting moment that is counterclockwise is needed for static equilibrium.

Always review your answer (all aspects: magnitude, direction, units, etc.) to determine if it makes sense relative to physically what you understand about the problem. Begin to think like an engineer.